Faculty of Engineering (Shoubra)
Engineering Mathematics and Physics Department
Mid term exam

Benha University
Mechanical Department
1st year Production
Time allowed: 1 hour

Student Name in Arabic: **Section:** **B.N.**

Correct the wrong statements giving the reason

a) If $\lim_{n\to\infty} U_n = 0$, then the Series $\sum_{n=1}^{\infty} U_n$ is convergent.

b) The Series $\sum_{n=1}^{\infty} P^n$ is convergent if $|P| < 1$.

c) The Series $\sum_{n=1}^{\infty} \frac{(-1)^n}{4^{2n+1}(n+1)}$ is conditionally convergent.

d) The differential equation $(y``)^2 + (y`)^5 + y = 0$ of order 2 and degree 5.

e) The envelope of straight line $x\cos\alpha + y\sin\alpha = P$ is a circle with center (α, α).

f) The minimal distance of the point (0,0,-1) from the plane given by $z=2x-y+1$ is (-2/3, 1/3, 4/3).

g) The integrating factor of the differential equation $xy\,dx + (x^2 + y)\,dy = 0$ is x.

Answer

a) Wrong, since the Series $\sum_{n=1}^{\infty} U_n$ may be divergent if $\lim_{n\to\infty} U_n = 0$, e.g. $\sum_{n=1}^{\infty} \frac{1}{n}$ is divergent and $\lim_{n\to\infty} \frac{1}{n} = 0$.

b) Correct, since $\sum_{n=1}^{\infty} P^n$ is convergent if $0<P<1$ while $\sum_{n=1}^{\infty} P^n$ is absolutely convergent if $-1<P<0$.

c)) Wrong , $U_n = \frac{1}{4^{2n+1}(n+1)}$ and $U_{n+1} = \frac{1}{4^{2n+3}(n+2)}$, therefore $U_n > U_{n+1}$ and $\lim_{n\to\infty} \frac{1}{4^{2n+1}(n+1)} = 0$ and $U_n = \frac{1}{4^{2n+1}(n+1)}$ is convergent, therefore $\sum_{n=1}^{\infty} \frac{(-1)^n}{4^{2n+1}(n+1)}$ is absolutely convergent.

d) Wrong, The D.E. $(y``)^2 + (y`)^5 + y = 0$ of order 2 and degree 2.

e) Wrong, since the envelope is $x^2 + y^2 = P^2$ which is circle with center (0,0).

f) Wrong, $f(x,y,z) = x^2 + y^2\,(z+1)^2$, $\phi(x,y,z) = z-2x+y= 1$ and $f_x = \lambda\,\phi_x$, $f_y = \lambda\,\phi_y$ and $f_z = \lambda\,\phi_z$, therefore $x = -2y$ and $z = y-1$, thus the nearest point is (-2/3, 1/3, -2/3) and the minimal distance is $\sqrt{\frac{2}{3}}$.

g) Wrong, The integrating factor of the differential equation $xy\,dx + (x^2 + y)\,dy = 0$ is y.

Dr. khaled Elnaggar

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a) If Sequence $\{U_n\}_{n=1}^{\infty}$ is convergent, then $\lim_{n\to\infty} U_n = 0$.

b) The Series $\sum_{n=1}^{\infty} P^{-n}$ is convergent if $|P| > 1$.

c) The Series $\sum_{n=1}^{\infty} \frac{2n(-1)^{n-1}}{4n^2-3}$ is divergent.

d) The differential equation $(y''')^2 + (y')^5 + y^7 = 0$ of order 7 and degree 3.

e) The envelope of circle $(x-\alpha)^2 + (y-\alpha)^2 = P^2$ is a line $x-y = P$.

f) A box having a square base and an open top is to contain 108 cubic feet with base area equal 16 square feet to obtain minimum area.

g) The integrating factor of the differential equation $\frac{dy}{dx} = -\frac{3xy+y^2}{x^2+xy}$ is y.

Answer

a) Wrong, sequence $\{U_n\}_{n=1}^{\infty}$ is convergent, then $\lim_{n\to\infty} U_n = L$, e.g. $\lim_{n\to\infty}(1+\frac{1}{n})^n = e \neq 0$ and $\{(1+\frac{1}{n})^n\}_{n=1}^{\infty}$ is convergent.

b) Correct, Since $\sum_{n=1}^{\infty}(\frac{1}{P})^n$ is convergent if P>1 and $\sum_{n=1}^{\infty}(\frac{1}{P})^n$ is absolutely convergent if P<-1.

c) Wrong, $U_n = \frac{2n}{4n^2-3}$, $U_{n+1} = \frac{2(n+1)}{4(n+1)^2-3}$, therefore $U_n > U_{n+1}$ and $\lim_{n\to\infty}\frac{2n}{4n^2-3} = 0$ and $U_n = \frac{2n}{4n^2-3}$ is divergent, thus $\sum_{n=1}^{\infty}\frac{2n(-1)^{n-1}}{4n^2-3}$ is conditionally convergent.

d) The differential equation $(y''')^2 + (y')^5 + y^7 = 0$ of order 3 and degree 2.

e) Wrong, since the envelope is pair of st. lines $x-y = \pm\sqrt{2}\,P$.

f) Wrong, $f(x,y,z) = x^2 + 4xy$, $\phi(x,y,z) = x^2y = 108$ and $f_x = \lambda\,\phi_x$, $f_y = \lambda\,\phi_y$ and $f_z = \lambda\,\phi_z$, therefore $\frac{1}{y} = \frac{2}{x}$, $x = 2y$, thus x=6, so the base area is 36.

g) Wrong, The integrating factor of the differential equation $\frac{dy}{dx} = -\frac{3xy+y^2}{x^2+xy}$ is x.

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a) If Series $\sum_{n=1}^{\infty} U_n$ is divergent, then $\lim_{n\to\infty} U_n \neq 0$.

b) The Series $\sum_{n=1}^{\infty} n^{-p}$ is convergent if P > 1.

c) The Series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} n^2}{n^2+3}$ is absolutely convergent.

d) The differential equation $y''' + (y')^5 + y^7 = 0$ of order 1 and degree 3.

e) The envelope of circle $(x-\alpha)^2 + y^2 = 2\alpha$ is a Parabola.

f) A box with largest volume and the total surface area is 64 cm^2, then the largest volume equal <u>32/3</u> cm^3.

g) The integrating factor of the differential equation $(x+2y^2)dx + 3xy\,dy = 0$ is x/3.

Answer

a) <u>Wrong,</u> if $\lim_{n\to\infty} U_n = 0$, the Series may be divergent

b) <u>Correct,</u> e.g. $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is convergent.

c) <u>Wrong,</u> $U_n = \frac{n^2}{n^2+3}$, $\lim_{n\to\infty} \frac{n^2}{n^2+3} = 1 \neq 0$, therefore $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} n^2}{n^2+3}$ is divergent.

d) D.E. $y''' + (y')^5 + y^7 = 0$ of order 3 and degree 1.

e) <u>Correct,</u> since the envelope is $y^2 = 2x + 1$

f) <u>Wrong,</u> $f(x,y,z) = xyz$, $\phi(x,y,z) = 2[xy+yz+xz] = 64$ and $f_x = \lambda\,\phi_x$, $f_y = \lambda\,\phi_y$ and $f_z = \lambda\,\phi_z$, therefore, $x = y = z = \sqrt{\frac{32}{3}}$, thus the largest volume $= \sqrt{(\frac{32}{3})^3}$.

g) <u>Wrong,</u> The integrating factor of the differential equation $\frac{dy}{dx} = -\frac{3xy+y^2}{x^2+xy}$ is $x^{1/3}$.

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a) If $\lim_{n\to\infty} U_n \neq 0$, then Series $\sum_{n=1}^{\infty} U_n$ may be divergent.

b) The Series $\sum_{n=1}^{\infty} n^{-p}$ is divergent if $P \leq 1$.

c) The Series $\sum_{n=1}^{\infty} (-\frac{1}{2})^n (1+\frac{1}{n})^{n^2}$ is absolutely convergent.

d) The differential equation $y''' + (y')^5 + y^7 = 0$ of order 1 and degree 3.

e) The envelope of the curve $2x + (y-\alpha)^2 = 2\alpha y$ is a Parabola.

f) An open box with volume 12 m^3, then the maximum area equal $12\sqrt[3]{9}$.

g) The integrating factor of the differential equation $(3xy+y^2)dx+(x^2+xy)dy = 0$ is ln x

Answer

a) Wrong, the Series $\sum_{n=1}^{\infty} U_n$ must be divergent.

b) Correct, e.g. $\sum_{n=1}^{\infty} \frac{1}{n}$ is divergent.

c) Wrong, $U_n = (\frac{1}{2})^n (1+\frac{1}{n})^{n^2}$ and $U_{n+1} = (\frac{1}{2})^{n+1} (1+\frac{1}{n+1})^{(n+1)^2}$

$U_n > U_{n+1}$ and $\lim_{n\to\infty} (\frac{1}{2})^n (1+\frac{1}{n})^{n^2} = 0$ and $U_n = (\frac{1}{2})^n (1+\frac{1}{n})^{n^2}$ is divergent, therefore $\sum_{n=1}^{\infty} (-\frac{1}{2})^n (1+\frac{1}{n})^{n^2}$ is conditionally convergent

d) D.E. $y''' + (y')^5 + y^7 = 0$ of order 3 and degree 1.

e) Correct, since the envelope is $3y^2 = 2x$.

f) Correct, $f(x,y,z) = 2[xy+yz+xz]$, $\phi(x,y,z) = xyz=12$, and $f_x = \lambda \phi_x$, $f_y = \lambda \phi_y$ and $f_z = \lambda \phi_z$, therefore, $x = y = 2z = 2\sqrt[3]{3}$, thus the maximum area = $12\sqrt[3]{9}$.

g) The integrating factor of the differential equation $(3xy+y^2)$

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	$dx+(x^2+xy)\,dy = 0$ is x