Benha University Faculty of Engineering- Shoubra Energy and Sustainable Energy program	Final Exam Date: 22 - 1 -2015 Mathematics 2 Code: EMP102 Duration : **3 hours**

Answer the following questions

1) Find k such that $2x^2 + kxy + 3y^2 + 5x - 5y + 2 = 0$ represent pair of straight lines, then find angle between the two lines and bisector lines.

2) Find the equation of the circle which intersects the circles $x^2 + y^2 + 2x - 2y + 1 = 0$ and $x^2 + y^2 + 4x - 4y + 3 = 0$ orthogonally and whose center lies on the line $3x - y - 2 = 0$.

3) Find equation of parabola whose focus is (1,2) and directrix $x+3y = 7$

4) Find equation of tangent to parabola $x^2 - 8x - 8y + 8 = 0$ at (8, 1).

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5) Evaluate the following integrals

a) $\int \frac{(x+4)dx}{\sqrt{x^2+6x+10}}$ b) $\int \frac{dx}{(x^2+4x+5)^2}$ c) $\int \frac{x^2+5}{x^2+3x+2}dx$

6) Solve the differential equation $y'' = x + \cos x$, $y(0)=2$, $y'(0)=4$

7) Find area bounded by two curves $f(x) = x$ and $g(x) = x^2$

8) Compute the arc length of the graph of $f(x)= x^{3/2}$ over [0, 1]

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Model answer

The above equation represents pair of straight lines if $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$ and from equation (12), a = 2, h = k/2, b = 3, g = 5/2, f = -5/2, c = 2, therefore $\begin{vmatrix} 2 & k/2 & 5/2 \\ k/2 & 3 & -5/2 \\ 5/2 & -5/2 & 2 \end{vmatrix} = 0 \Rightarrow$ 2[6-25/4] – (k/2)[k + 25/4] + (5/2)[[-5k/4 – 15/2] = 0 $\Rightarrow$ $2k^2 + 25k + 77 = 0 \Rightarrow$ (2k + 11)(k + 7) = 0 $\Rightarrow$ k = -11/2 or k = -7.

At k = -7, the equation of the pair of st. lines is:

$2x^2 - 7xy + 3y^2 + 5x - 5y + 2 = 0 \Rightarrow (2x - y + c_1)(x - 3y + c_2) = 0$, therefore $2x^2 - 7xy + 3y^2 + (2c_2 + c_1)x - (3c_1 + c_2)y + c_1c_2 = 0$ and by comparing coefficients of x and y such that:

$$2c_2 + c_1 = 5, \quad 3c_1 + c_2 = 5$$

By solving the two equations simultaneously, we get $c_1 = 1$ and $c_2 = 2$ and therefore the two lines are 2x – y + 1 = 0, and x – 3y + 2 = 0.

Since a = 2, h = -7/2, b = 3, $\tan\theta = \dfrac{2\sqrt{(-7/2)^2 - (2)(3)}}{2+3} = 1$, therefore $\theta = \tan^{-1}(1) = \dfrac{\pi}{4}$.

The bisector lines are $\dfrac{2x - y + 1}{\sqrt{4+1}} = \pm\dfrac{x - 3y + 2}{\sqrt{1+9}}$

At k = -11/2, the equation of the pair of st. lines is:

$4x^2 - 11xy + 6y^2 + 10x - 10y + 4 = 0 \Rightarrow (4x - 3y + c_1)(x - 2y + c_2) = 0$, therefore $4x^2 - 11xy + 6y^2 + (4c_2 + c_1)x - (2c_1 + 3c_2)y + c_1c_2 = 0$ and by comparing coefficients of x and y such that:

$$4c_2 + c_1 = 10, \quad 2c_1 + 3c_2 = 10$$

By solving the two equations simultaneously, we get $c_1 = 1$ and $c_2 = 2$ and therefore the two lines are 4x – 3y + 1 = 0, and x–2y + 2 = 0.

Since a = 2, h = -11/4, b = 3, $\tan\theta = \frac{2\sqrt{(-11/4)^2-(2)(3)}}{2+3}$ = 1, therefore $\theta = \tan^{-1}(1) = \frac{\pi}{4}$.

The bisector lines are $\frac{4x-3y+1}{\sqrt{16+9}} = \pm\frac{x-2y+2}{\sqrt{1+4}}$

2) Let the equation of the required circle be $x^2 + y^2 + 2gx + 2fy + c = 0$ ...(i)

Since this circle cuts the circles $x^2 + y^2 + 2x - 2y + 1 = 0$ and $x^2 + y^2 + 4x - 4y + 3 = 0$ orthogonally, we get 2(g. 1 + f. (-1)) = c +1 => 2 g -2 f - c -1 = 0 ...(ii)

And 2(g(2) + f(-2)) = c +3 => 4 g - 4f - c - 3 = 0 ...(iii)

As center of (i) i.e. (-g, -f) lies on 3x - y - 2 = 0, we get

-3g + f - 2 = 0 => 3g - f + 2 = 0 ...(iv)

Subtracting (ii) from (iii), so 2g -2f -2 = 0 => g -f -1 = 0 ...(v)

Solving (iv) and (v) simultaneously, then g= -3/2 and f = -5/2

From (ii), we get c = 2 g -2 f -1 = -3 +5 -1 = 1 and Substituting these values of g, f and c in (i), we get $x^2 + y^2 - 3x - 5y + 1 = 0$, which is the equation of the required circle.

3) Let P(x, y) is a point on parabola such that $\sqrt{(x-1)^2+(y-2)^2} = \frac{|x+3y-7|}{\sqrt{1^2+3^2}}$

By squaring, we get $9x^2-6xy+y^2-6x+2y+1=0$, which is the equation of parabola.

4) By completing the square, we get $(x-4)^2 = 8(y+1)$, then p = 2 and h = 4, k = -1, $x_1 = 8$ and $y_1 = 1$, thus the equation of tangent is $(x_1 - h)(x - h) = 2p(y + y_1 - 2k) \Rightarrow 4(x-4) = 4(y+3)$

5-a) By completing square, we get $\int \frac{(x+4)dx}{\sqrt{x^2+6x+10}} = \int \frac{(x+3+1)dx}{\sqrt{(x+3)^2+1}}$

$$= \frac{1}{2}\int \frac{2(x+3)dx}{\sqrt{(x+3)^2+1}} + \int \frac{dx}{\sqrt{(x+3)^2+1}} = \sqrt{(x+3)^2+1} + \sinh^{-1}(x+3)$$

5-b) put x+2 = $\tan\theta \Rightarrow dx = \sec^2\theta \, d\theta$, therefore $\int \frac{dx}{(x^2+4x+5)^2} = \int \frac{dx}{[(x+2)^2+1]^2} =$

$$\int \frac{\sec^2\theta d\theta}{[(\tan\theta)^2+1]^2} = \int \cos^2\theta d\theta = \frac{1}{2}\int [1+\cos 2\theta]d\theta = \frac{\theta + \sin\theta\cos\theta}{2}$$

$$= \frac{\tan^{-1}(x+2)}{2} + \frac{(x+2)}{2\sqrt{x^2+4x+5}}$$

5-c) The degree of numerator must be less than that of denominator, so we have to make long division

$$\begin{array}{r|l} & 1 \\ x^2+3x+2 & x^2+5 \\ & \underline{x^2+3x+2} \\ & -3x+3 \end{array} \qquad \frac{x^2+5}{x^2+3x+2} = 1 + \frac{-3x+3}{x^2+3x+2}$$

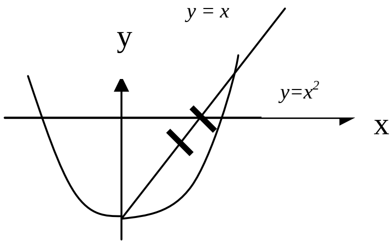
$$\int \frac{x^2+5}{x^2+3x+2}dx = \int \left(1+\frac{-3x+3}{x^2+3x+2}\right)dx = \int \left(1+\frac{-3x+3}{(x+2)(x+1)}\right)dx = x + \int \left(\frac{A}{(x+1)}+\frac{B}{(x+2)}\right)dx$$

By using partial fraction, we will get A= 6,   B = -9

$$\int \frac{x^2+5}{x^2+3x+2}dx = x + \int \left(\frac{6}{(x+1)} - \frac{9}{(x+2)}\right)dx = x + 6Ln(x+1) - 9Ln(x+2) + c$$

6- y\`(x)= $x^2/2$ + sinx +c , but at y\`=4, x = 0, therefore c = 4, therefore y\`(x)= $x^2/2$ + sinx +4, integrate, we will get y(x)= $x^3/6$ – cosx + 4x+d , but at y=2 , x=0, therefore d= 3, thus the solution is y(x)= $x^3/6$ – cosx + 4x+3

7) $A = \int_0^1 [x - x^2]dx = \left.\frac{x^2}{2} - \frac{x^3}{6}\right|_0^1 = \frac{1}{2} - \frac{1}{6} = \frac{1}{3}$



8) f\`(x) = (3/2) $x^{1/2}$, therefore $\mathbf{L = \int_0^1 \sqrt{1+[f`(x)]^2}dx = \int_0^1 \sqrt{1+\frac{9}{4}x}\,dx}$

$$= \frac{8}{27}(1+\frac{9}{4}x)^{3/2}\Big|_0^1 = (\frac{13}{4})^{3/2} - 1$$