Benha University Faculty of Engineering- Shoubra Mechanical Engineering Department Second year Mech. Eng(Production)		Final term exam Date: 14/1/2016 Code: EMP 291 Engineering Mathematics Duration : 3 hours
Answer all the following questions	No. of questions : **Three**	Total Mark: **80**

Question 1 **[30 marks]**

(a) Show that $v(x,y) = e^{2x}\cos 2y$ is harmonic and find $u(x,y)$ such that $f(z) = u + iv$ is analytic. **[10 marks]**

(b) Evaluate the following integrals: **[15 marks]**

$$(i)\int_c (z^2+\cos z)^{20}\,dz\,,\ |z-8| = 10$$

$$(ii)\int_c \frac{4-3z}{z(z-2)}\,dz \quad ;\ c\ is\ \left|z-\frac{1}{2}\right| = \frac{1}{2}$$

$$(iii)\int_c \frac{\cos z}{z+\pi} \quad ;\ c\ is\ |z+3| = 2$$

(c) Deduce Laplace transform of unit step function **[5 marks]**

Question 2 **[30 marks]**

(a) Solve the following differential equation using Laplace transform: **[10 marks]**

$$y'' + 2y' + y = e^{-t}\ ;\quad y(0) = 0\ ;\ y'(0) = 1$$

(b) By using Euler's method solve the I.V.P $y' = xy + 1;\ y(0) = 1$ **[10 marks]**
to get y (0.6) with h= 0.2

(c) Find a cubic interpolation polynomial which interpolate the function **y= f(x)** at the points (1, -4) , (2,8) , (5,140) , (8,542) by divided difference method **[10 marks]**

Question 3 **[20marks]**

(a) Fit the function $y = a\sin x + b$ that best fit the data (1,0), (2,3) , (5,1) **[10 marks]**

(b) solve the heat equation $\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x^2}$, $t > 0$ **[10 marks]**
by separation method, subject to $u(0,t) = 0, u(x,0) = 9\sin 2x$

Good Luck *Dr.Eng. Zaki Ahmed Zaki*

Second year Mech. Eng. (production) final term Exam (Jan. 2016)

Model Answer

Question 1 (a) Show that $v(x,y) = e^{2x}\cos 2y$ **is harmonic and find** $u(x,y)$ **such that** $f(z) = u + iv$ **is analytic,** **[10 marks]**

$v_x = 2e^{2x}\cos 2y$,$v_{xx} = 4e^{2x}\cos 2y$,$v_y = -2e^{2x}\sin 2y$,$v_{yy} = -4e^{2x}\cos 2y$

$\because v_{xx} + v_{yy} = 0$; *then v is harmonic*

$\because f(z)$ *is analytic then* $u_x = v_y \rightarrow u = \int v_y\, dx = \int -2e^{2x}\sin 2y\, dx$

$u = -e^{2x}\sin 2y + c(y)$; $\because u_y = -v_x \rightarrow -2e^{2x}\cos 2y + c'(y) = -2e^{2x}\cos 2y$

$c'(y) = 0 \rightarrow c(y) = cons\tan t$ *may be neglected*

$\therefore u = -e^{2x}\sin 2y$

$f(z) = u + iv = -e^{2x}\sin 2y + i\, e^{2x}\cos 2y$

(b) Evaluate the following integrals: **[15 marks]**

(i) $\oint_c (z^2 + \cos z)^{20}\, dz \quad ;\ c\ is\ |z - 8| = 10$

The function is analytic (defined) inside the region then

$\oint_c (z^2 + \cos z)^{20}\, dz = 0$

$(ii) \int_c \frac{4 - 3z}{z(z-2)}\, dz \quad ;\ c\ is\ \left|z - \frac{1}{2}\right| = \frac{1}{2}$

Z=0 inside but z =2 outside

$$\int_c \frac{4-3z}{z(z-2)}\, dz = \int_c \frac{\frac{4-3z}{(z-2)}}{z}\, dz = 2\pi i\ \frac{4}{-2} = -4\pi i$$

$(iii) \int_c \frac{\cos z}{z + \pi} dz \quad ;\ c\ is\ |z + 3| = 2$

$z = -\pi\ inside\ |z + 3| = 2\ then,$

$$= \int_c \frac{\cos z}{z + \pi} dz = 2\pi i\ \cos(-\pi) = -2\pi i$$

(c) Deduce Laplace transform of unit step function **[5 marks]**

$$u(t-a) = \begin{cases} 1 & ;t \succ a \\ 0 & ;t \prec a \end{cases} \quad ; L[u(t-a)] = \int_0^\infty u(t-a)e^{-st}dt = \int_a^\infty 1e^{-st}dt = -\frac{1}{s}[e^{-\infty} - e^{-as}] = \frac{1}{s}e^{-as}$$

Question 2 **[30 marks]**

(a) Solve the following using Laplace transform: [10 marks]

$$y'' + 2y' + y = e^{-t} \quad ; \quad y(0) = 0 \quad ; \ y'(0) = 1$$

$$y'' + 2y' + y = e^{-t} \;;\quad y(0) = 0 \;;\; y'(0) = 1$$

$$s^2 Y(s) - sy(0) - y'(0) + 2sY(s) - 2y(0) + Y(s) = \frac{1}{s+1}$$

$$Y(s)(s^2 + 2s + 1) = 1 + \frac{1}{s+1} \quad then\, Y(s) = \frac{1}{(s+1)^2} + \frac{1}{(s+1)^3}$$

$$y(t) = t e^{-t} + \frac{t^2}{2!} e^{-t}$$

(b) By using Euler's method solve the I.V.P [10 marks]

$y' = xy + 1;\; y(0) = 1$ to get $y(0.6)$ with $h = 0.2$

$$y' = 1 + xy \,; y_{i+1} = y_i + 0.2(1 + x_i y_i), x_0 = 0, y_0 = 1$$

$$y_1 = y_0 + 0.2(1 + x_0 y_0) = 1.2, y_2 = 1.448, y_3 = \boxed{y(0.6) = 1.7638}$$

(c) [10 marks]

$f(x) = -4 + 12(x-1) + (x-1)(x-2)8 + (x-1)(x-2)(x-5)7 = x^3 + 5x - 10$

x	y			
1	-4	12	8	
2	8	44	15	1
5	140	134		
8	542			

Q3 [20 marks]

[a] Fit the function $y = a \sin x + b$ that best fit the data (1,0), (2,3) , (5,1) [10 marks]

$Y = aX + b$,

$$\sum Y = a \sum x + 3b, and \;\; \sum XY = a \sum x^2 + b \sum X$$

x	y	X= sin x	X^2	Xy
1	0	0.841	0.707	0
2	3	0.91	0.828	2.73
5	1	-0.959	0.92	-0.959
sum	4	0.792	2.455	1.771

4=0.792a+3b , 1.771=2.455a+0.792b ,,Then **a = 0.318 , b=1.249** ***then***

$$y = 0.318 \sin x + 1.249$$

(b) [10 marks]

$$u(x,t) = F_{(x)} G_{(t)}; FG' = 2F''G \;; \frac{F''}{F} = \frac{G'}{2G} = -\alpha^2$$

$$F = c_1 \cos\alpha x + c_2 \sin\alpha x; G = A e^{-2\alpha^2 t}$$

$$u(x,t) = e^{-2\alpha^2 t}[B\cos\alpha x + D\sin\alpha x]; u(0,t) = 0 then B = 0,$$

$$u(x,t) = e^{-2\alpha^2 t} D \sin\alpha x$$

$$u(x,0) = D\sin\alpha x = 9\sin 2x, then\, D = 9 AND\, \alpha = 2$$

$$u(x,t) = 9e^{-8t} \sin 2x$$

Good luck *Dr.Eng. Zaki A. Zaki*