Benha University
Faculty of Engineering- Shoubra
Credit hour system Programs
Final Examination

Subject: Mathematics 1
Code: EMP 101
Date: 23/1/2016
Duration : 3 hours

- **Answer all the following questions**
- **Illustrate your answers with sketches when necessary.**
- **The Exam. Consists of one Page**
- **No. of Questions: 2**
- **Total Mark:** 40 Marks

Question (1):

a) Evaluate the following limits i) $\lim_{x\to\infty}\left[\frac{x+3}{x-7}\right]^{2x}$ ii) $\lim_{x\to 0}[\frac{1}{e^x-1}]^{\tan x}$ [8]

b) Find the first derivative for the following functions [12]

i) $y = \tan[(\sec x)^{\cos x}] + \ln(\arcsin x^3)^{\tan x}$

ii) $y = \sqrt{1+\sqrt{1+\sqrt{x}}} + \frac{4^{\csc 3x}\tan(\ln x)}{e^{\sin x}}$

Question (2)

a) Expand $(3 - x^2)^{-1/5}$, then find the general term [5]

b) Resolve $\frac{x^4+3x^2+x+1}{(x+2)^2(x-3)}$ into partial fraction [5]

c) Express $\cos 3\theta$ in terms of $\cos\theta$ only using Demoivre theorem [5]

d)Solve the system of equations $x - 3z = -20$, $y - x + 2z = 17$, $2x + y = 6$ using Gauss Jordan method. [5]

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Answer of question 1

a-i) $\lim_{x\to\infty}\left[\frac{x+3}{x-7}\right]^{2x} = \lim_{x\to\infty}\left[\frac{x-7+10}{x-7}\right]^{2x} = \lim_{x\to\infty}\left[1+\frac{10}{x-7}\right]^{2x}$

Put x-7 =y, therefore $\lim_{x\to\infty}\left[\frac{x+3}{x-7}\right]^{2x} = \lim_{y\to\infty}\left[1+\frac{10}{y}\right]^{2(y+7)} = e^{20}$

ii) put y =$[\frac{1}{e^x-1}]^{\tan x}$, hence ln y = -tanx ln($e^x$ – 1), thus $-\lim_{x\to 0}\tan x\ln(e^x-1)$ of type (0. ∞)

Therefore $-\lim_{x\to 0}\tan x\ln(e^x-1) = -\lim_{x\to 0}\frac{\ln(e^x-1)}{1/\tan x} = \lim_{x\to 0}\frac{e^x/(e^x-1)}{\csc^2 x}$

$= \lim_{x\to 0}\frac{e^x\sin^2 x}{e^x-1} = \lim_{x\to 0}\frac{e^x\sin^2 x + 2e^x\sin x\cos x}{e^x} = 0$,thus $\lim_{x\to 0}[\frac{1}{e^x-1}]^{\tan x} = 1$

b) $y = \tan[(\sec x)^{\cos x}] + \tan x\ln(\arcsin x^3)$, therefore

$y` = [\sec^2[(\sec x)^{\cos x}]](\sec x)^{\cos x}[-\sin x\ln\sec x + \frac{\sec x\tan x}{\sec x}\cos x] +$

$\sec^2 x\ln(\arcsin x^3) + \tan x[\frac{3x^2/[\sqrt{1-x^6}}{\arcsin x^3}]$

$=[\sec^2[(\sec x)^{\cos x}]](\sec x)^{\cos x}[-\sin x\ln\sec x + \sin x]+$

$\sec^2 x\ln(\arcsin x^3) + \tan x[\frac{3x^2/[\sqrt{1-x^6}}{\arcsin x^3}]$

ii) $y` = \frac{1}{2}[1+[1+x^{1/2}]^{1/2}]^{-1/2}[\frac{1}{2}[1+x^{1/2}]^{-1/2}][\frac{1}{2}x^{-1/2}] +$

$$\frac{([-3\csc 3x\cot 3x\, 4^{\csc 3x}\ln 4]\tan(\ln x) + 4^{\csc 3x}\frac{\sec^2 \ln x}{x})e^{\sin x} - 4^{\csc 3x}\cos x\, e^{\sin x}}{e^{2\sin x}}$$

Answer of question 2

a) $(3 - x^2)^{-1/5} = 3^{-1/5}(1-\frac{x^2}{3})^{-1/5} = 3^{-1/5}(1 - \frac{1}{5}(-\frac{x^2}{3}) + \frac{\frac{1}{5}(\frac{6}{5})(-\frac{x^2}{3})^2}{2!} - \frac{\frac{1}{5}(\frac{6}{5})(\frac{7}{5})(-\frac{x^2}{3})^3}{3!} - \ldots$

The general term is $\frac{(-1)^r \frac{1}{5}(\frac{6}{5})....(\frac{1}{5} + r - 1)(-\frac{x^2}{3})^r}{r!}$

b) By long devision,

$\frac{x^4 + 3x^2 + x + 1}{(x+2)^2(x-3)} = x - 1 + \frac{12x^2 + 5x - 11}{(x+2)^2(x-3)} = x - 1 + \frac{A}{x+2} + \frac{B}{(x+2)^2} + \frac{c}{x-3}$, so that

such that $A(x+2)(x-3) + B (x-3) + c (x+2)^2 = 12x^2 + 5x - 11$

At $x = -2$, $B = -27/5$ and at $x = 3$, $c = 112/25$ and finally at $x = 0$, therefore

$-6 A - 3B + 4c = -11$, fro which we get A

c)

$\cos 3\theta + i\sin 3\theta = (\cos\theta + i\sin\theta)^3 = (\cos\theta)^3 + 3(\cos\theta)^2(i\sin\theta) + 3(\cos\theta)(i\sin\theta)^2 + (i\sin\theta)^3$

Thus $\cos 3\theta = (\cos\theta)^3 - 3(\cos\theta)(\sin\theta)^2 = (\cos\theta)^3 - 3(\cos\theta)(1-\cos^2\theta) = 4(\cos\theta)^3 - 3\cos\theta$

3- Intended Learning Outcomes of Course (ILOS)

a- Knowledge and Understanding

On completing this course, students will be able to:

a.1 - Recognize concepts and theories of Differentiation and Integration: Functions, Limits, Differentiation Indefinite integrals, Integral properties, Linear Algebra: Binomial Theorem, Matrices.(A.1)

a.2- Solving engineering problems, data collection interpretation for Differentiation and Integration (A.5)

b- Intellectual Skills

At the end of this course, the students will be able to:

b.1 - Select appropriate mathematical methods for analyzing of Differentiation and Integration problems (B.1)

b.2 - Select appropriate solutions for engineering problems based on Differentiation and Integration concepts and theories, Matrix properties(B.2)

b.3 - Solve engineering problems, often on the basis of Differentiation and Integration, Linear Equations and Matrices(B.7)

c- Professional Skills

On completing this course, the students are expected to be able to:

c.1 - Apply knowledge of mathematics,Differentiation and Integration, Linear EquationsMatrices (C.1)

c.2 - Apply numerical modeling methods and/or appropriate computational techniques to Differentiation and Integration engineering problems (C.7)

d- General Skills

At the end of this course, the students will be able to:

d.1- Communicate effectively (D.3)

d.2-Engage in self-learning and improving performance (D.7)

Questions	Total marks	Achieved ILOS
Q1	20	b1,a2,c1
Q2	20	a1,b1,b2