Benha University
Faculty of Engineering at Shoubra
Electrical Engineering Department
Postgraduate (M.Sc.)
Communication Engineering

Final Term Exam
Date: Sunday 5/1/2016
Subject: Digital Signal Processing
Duration: 3 hours

- **Answer all the following questions**
- **Illustrate your answers with sketches when necessary**
- **No. of questions : 4**
- **Total Mark: 210 Marks**

Model Answer

Question (1) Systems and LTI [80 Marks]:

1. Which of the following system is linear? **[30 Marks]**

a) Differentiator y[n] = dx[n]/dn

Answer

For x1[n] : $y1[n] = \frac{dx1[n]}{dn}$

For x2[n] : $y2[n] = \frac{dx2[n]}{dn}$

For x1[n] + x2[n] : $y'[n] = \frac{d\{x1[n]+x2[n]\}}{dn} = \frac{dx1[n]}{dn} + \frac{dx2[n]}{dn}$

$= y1[n] + y2[n]$

Therefore, the differentiator is a linear system.

b) Amplifier y[n]= 5 x[n]

Answer

For x1[n] : $y1[n] = 5\,x1[n]$

For x2[n] : $y2[n] = 5\,x2[n]$

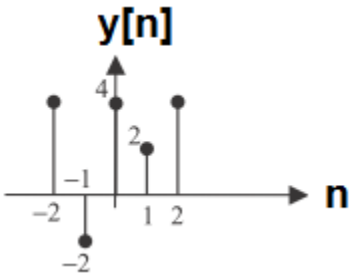
For x1[n] + x2[n] :

$y'[n] = 5\{x1[n] + x2[n]\} = 5\,x1[n] + 5\,x2[n] = y1[n] + y2[n]$

Therefore, the amplifier is a linear system.

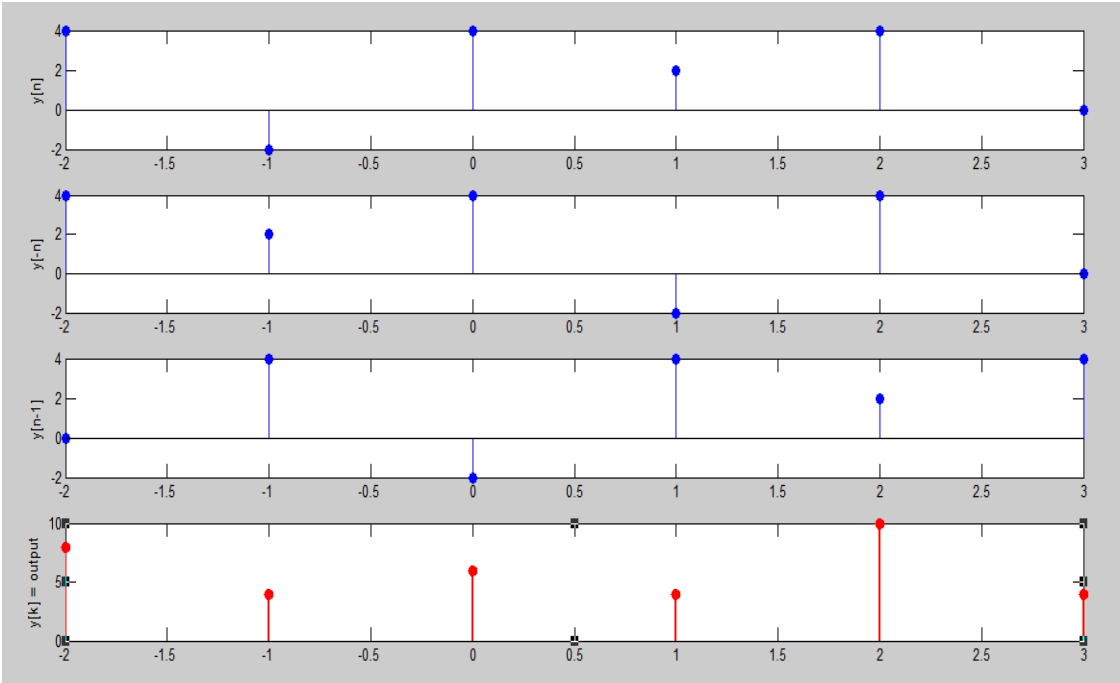
2. **The output h[k] of a DT LTI system in response to a unit impulse function δ[k] is shown in Figure. Find the output for the following input:**

x[k] = δ[-k] + δ[k] + δ[k-1] **[30 Marks]**



Answer

- **The given figure is the response of the system to an impulse function δ[k]**
- **Using the linearity and shift properties of the given LTI system , we can find the output as a combination of the shifted versions of the given output:**
  Y[k] = y[-n] + y[n] + y[n-1] = [8 4 6 4 10 4]
  The combination is given in the next figure



It can be solved using convolution too

3. Describe in details the structure of the LTI systems for both Infinite-Length and Finite-Length Signals, and what is the type of the discrete -system matrix (H). [20 Marks]

Answer

LTI Systems are Toeplitz Matrices (Infinite-Length Signals) (1)

- For an LTI system with infinite-length signals

$$h_{n,m} = h_{n+q,m+q} \quad \forall\, q \in \mathbb{Z}$$

$$\mathbf{H} = \begin{bmatrix} & \vdots & \vdots & \vdots & \\ \cdots & h_{-1,-1} & h_{-1,0} & h_{-1,1} & \cdots \\ \cdots & h_{0,-1} & h_{0,0} & h_{0,1} & \cdots \\ \cdots & h_{1,-1} & h_{1,0} & h_{1,1} & \cdots \\ & \vdots & \vdots & \vdots & \end{bmatrix} = \begin{bmatrix} & \vdots & \vdots & \vdots & \\ \cdots & h_{0,0} & h_{-1,0} & h_{-2,0} & \cdots \\ \cdots & h_{1,0} & h_{0,0} & h_{-1,0} & \cdots \\ \cdots & h_{2,0} & h_{1,0} & h_{0,0} & \cdots \\ & \vdots & \vdots & \ddots & \end{bmatrix}$$

- Entries on the matrix diagonals are the same – **Toeplitz matrix**

LTI Systems are Circulent Matrices (Finite-Length Signals) (1)

- For an LTI system with length-N signals

$$h_{n,m} = h_{(n+q)_N,(m+q)_N} \quad \forall\, q \in \mathbb{Z}$$

$$\begin{bmatrix} h_{0,0} & h_{0,1} & h_{0,2} & \cdots & h_{0,N-1} \\ h_{1,0} & h_{1,1} & h_{1,2} & \cdots & h_{1,N-1} \\ h_{2,0} & h_{2,1} & h_{2,2} & \cdots & h_{2,N-1} \\ \vdots & \vdots & \vdots & & \vdots \\ h_{N-1,0} & h_{N-1,1} & h_{N-1,2} & \cdots & h_{N-1,N-1} \end{bmatrix} = \begin{bmatrix} h_{0,0} & h_{N-1,0} & h_{N-2,0} & \cdots & h_{1,0} \\ h_{1,0} & h_{0,0} & h_{N-1,0} & \cdots & h_{2,0} \\ h_{2,0} & h_{1,0} & h_{0,0} & \cdots & h_{3,0} \\ \vdots & \vdots & \vdots & & \vdots \\ h_{N-1,0} & h_{N-2,0} & h_{N-3,0} & \cdots & h_{0,0} \end{bmatrix}$$

- Entries on the matrix diagonals are the same + circular wraparound – **circulent matrix**

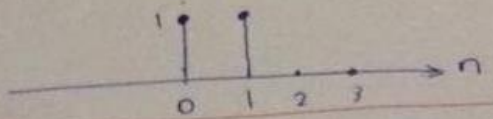
Question (2) Convolution [25 Marks]:

a) **Find y[n] = x[n]*h[n], where:**
   x[n] = u[n] –u[n-2] and
   h[n] = u[n] –u[n-4]
b) **Determine the convolution duration.**

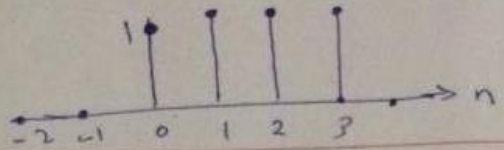
Answer

- **The length of signals:**
  Nx = 2
  Nh =4
  Ny = convolution duration = Nx + Nh -1
  = 2+4-1 = 5 samples
- **By applying convolution:**
  First flip x[n] and shift as in the following figures:

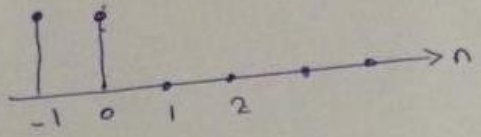
- $X[n] = u[n] - u[n-3]$



- $h[n] = u[n] - u[n-4]$

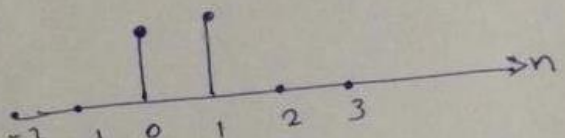


- $X[-n]$



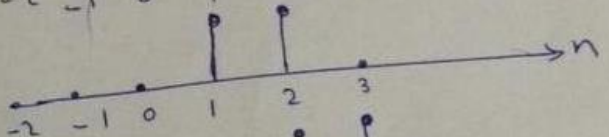
$\Rightarrow y[0] = 1$

- $X[-n-1]$



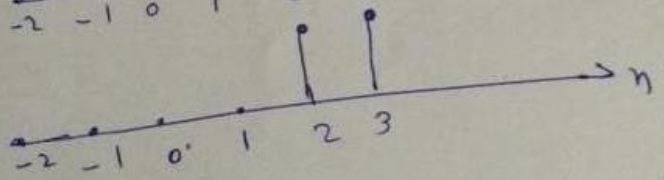
$\Rightarrow y[1] = 2$

$X[-n-2]$



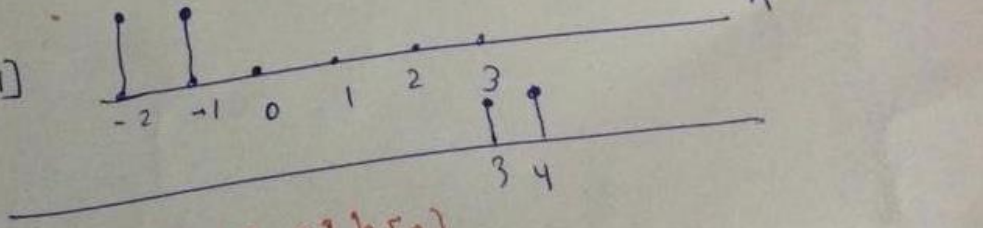
$\Rightarrow y[2] = 2$

$X[-n-3]$



$\Rightarrow y[3] = 2$

$X[n+1]$



$\Rightarrow y[4] = 1$

$$X[n] * h[n] = \sum X[n-k]\, h[n]$$

$$y[n] = [1\ 2\ 2\ 2\ 1]$$