Benha University Faculty of Engineering- Shoubra Engineering Mathematics and Physics Department		Final Term Exam Date:5 /1/ 2016 Numerical analysis (EMM 404) Time: 3 hours (تأهيلي ماجستير رياضيات هندسية)
Answer all the following questions	No. of questions : **Two**	Total Mark: **200**

Question 1 **[100 marks]**

(a) Find a cubic interpolation polynomial which interpolate the function y = **f(x)** at the points (1,9) , (2,26) , (3,55) ,(4,102) .Hence find the value of x which makes f**(x) = 0** by fixed method . **[40 marks]**

(b) Fit the function $y = a\sin^2 x + b$ that best fit the data $(0, 4.2)$, $(\frac{\pi}{6}, 5.8)$, $(\frac{\pi}{4}, 8.3)$, $(\frac{\pi}{2}, 12.5)$ **[20 marks]**

(c) By using **Euler's method** solve $y' = xy + 1; y(0) = 1$ to get $y(0.6)$ with $h = 0.2$ **[20 marks]**

(d) Deduce A, B, C such that $\acute{f}(x) = Af(x) + Bf(x-h) + Cf(x-2h)$ **[20 marks]**

Then find $\acute{f}(1.4)$ from the data (1, -1.718) , (1.2, -2.120), (1.4, -2.655)

Question 2 **[100 marks]**

(a) Use the following data to approximate the integration of the function **f(x)** from 1 to 5 By Simpson's rule **[30 marks]**

x	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**	**4.5**	**5**
f(x)	**0**	**0.41**	**0.69**	**0.92**	**1.10**	**1.25**	**1.39**	**1.5**	**1.61**

(**b**) Given the data: **[30 marks]**

x	**0.2**	**0.4**	**0.6**	**0.8**	**1.00**	**1.2**
y	**0.163746**	**0.268128**	**0.329287**	**0.359463**	**0.367879**	**0.361433**

Find $f''(0.6)$. If the data are taken for the function $f(x) = xe^{-x}$ Calculate the error

(c) Given the heat equation $\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x^2}; t \succ 0$ **[40 marks]**

subject to $u(0,t) = 0, u(3,t) = 0$ and $u(x,0) = x(x-3)$

Using the finite difference scheme to compute **u (x =1 , t = 1)** take h= 0.5 ; k =0. 5

Good Luck *Dr.Eng. Zaki Ahmed Zaki*

Final term Exam (5/1/2016) (تأهيلي ماجستير رياضيات هندسية)

Model Answer

Question 1 [100 marks]

(a) Find a cubic interpolation polynomial which interpolate the function y = **f(x)** at the points (1,9) , (2,26) , (3,55) ,(4,102) .Hence find the value of x which makes f**(x) = 0** by fixed method . **[40 marks]**

x	y			
1	9	17	12	
2	26	29	18	6
3	55	47		
4	102			

By Newton forward: y(x)=9+(x-1)17+(x-1)(x-2)12/2 ++(x-1)(x-2)(x-3)6/6

$y(x)= \underline{x^3 +10x -2}$,,,by fixed $x_{n+1}=(2- x_n^3)/10$,,,root in (0,1)

i	x_{n+1}=(root),, x_0=0.5
0	0.1875
1	0.19934
2	0.1992078
3	0.1992094

Root=0.1992094

(b) Fit the function $y = a\sin^2 x + b$ that best fit the data **[20 marks]**
(0, 4.2) , ($\frac{\pi}{6}$, 5.8) , ($\frac{\pi}{4}$, 8.3), ($\frac{\pi}{2}$, 12.5)

30.8=1.75a+4b and 18.1=1.3125a+1.75b then a=8.457 , b=4

x	y	X	Xy	X^2
0	4.2	0	0	0
$\frac{\pi}{6}$	5.8	0.25	1.45	0.0625
$\frac{\pi}{4}$	8.3	0.5	4.15	0.25
$\frac{\pi}{2}$	12.5	1	12.5	1
SUM	**30.8**	**1.75**	**18.1**	**1.3125**

(c) By Euler's method: **[20 marks]**

$y' = xy + 1; y_{n+1} = y_n + 0.2(1 + x_n y_n), y_0 = 1, x_0 = 0, x_1 = 0.2, x_2 = 0.4, x_3 = 0.6$

$y_1 = y_0 + 0.2(1 + x_0 y_0) = y(0.2) = 1.2,$

$y_2 = y_1 + 0.2(1 + x_1 y_1) = y(0.4) = 1.448,$

$y_3 = y_2 + 0.2(1 + x_2 y_2) = \boxed{y(0.6) = 1.7638}$

(**d**)Deduce A, B, C such that $\acute{f}(x) = Af(x) + Bf(x-h) + Cf(x-2h)$ **[20 marks]**

Let f(x) =1 , x And x^2 then **A=3, B=-4 , C=1**

Then find $\acute{f}(1.4)$ from the data (1, -1.718) , (1.2, -2.120), (1.4, -2.655)

$$\acute{f}(1.4) = \frac{3(-2.655) - 4(-2.120) + (-1.718)}{2(0.2)} = -3.0085$$

Question 2 **[100 marks]**

(a) Use the following data to approximate the integration of the function **f(x)** from 1 to 5 By Simpson's rule **[30 marks]**

x	1	1.5	2	2.5	3	3.5	4	4.5	5
f(x)	0	0.41	0.69	0.92	1.10	1.25	1.39	1.5	1.61

x	f(x)	2f(x) or4f(x)
1	0	0.00
1.5	0.41	1.64
2	0.69	1.38
2.5	0.92	3.68
3	1.10	2.20
3.5	1.25	5.00
4	1.39	2.78
4.5	1.5	6.00
5	1.61	1.61
	sum	**24.29**

Simpson's method: $\int_1^5 f(x)\,dx = \frac{0.5}{3} 24.29 = 4.0483$

(b) By using five point formula: **[30 marks]**

$$f''(0.6) = \frac{-(0.367879) + 16(0.359463) - 30(0.329287) + 16(0.268128) - 0.1637460.329287}{12(0.2)^2} = \boxed{-0.7682895}$$

$$f(x) = xe^{-x} \rightarrow f'(x) = (1-x)e^{-x}$$

$$f''(x) = (x-2)e^{-x} \rightarrow f''(0.6) = -0.7683362$$

$$Error = \| -0.7683362 - (-0.7682895) \| = 4.67 * 10^{-5}$$

(c) Given the heat equation $\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x^2}; t \succ 0$ **[40 marks]**

subject to $u(0,t) = 0, u(3,t) = 0$ and $u(x,0) = x(x-3)$

Using the finite difference scheme to compute **u (x =1 , t = 1)** take h= 0.5 ; k =0. 5

$$r = \frac{ka}{h^2} = \frac{0.5(2)}{(0.5)^2} = 4; u_{i,j+1} = 4(u_{i+1,j} + u_{i-1,j}) - 7u_{i,j}$$

$$u_1 = 0.75, u_2 = 0, u_3 = -0.25, \boxed{u_9 = u(x=1, t=1) = 2}$$

Good Luck *Dr.Eng. Zaki Ahmed Zaki*