Benha University Faculty of Engineering – Shoubra Department of Energy and Sustainable Energy Course: Mathematics 1-A (Prep. Year)		Final Exam Date: January 2016 Duration: 3 hours Answer **All** questions

- The exam consists of one page
- No. of questions: 4 Total Mark: 40

Question 1 9

Find **y`** from the following:

(a) $y = x^5 + 2^{x^3} - x$ (b) $y = \cosh x^3 . \operatorname{sech} x$ (c) $y = \sin x^2 - \ln \cos x$

(d) $y = \tanh^{-1} x + \tan x$ (e) $\sqrt{xy} = \log(x + y)$ (f) $y = t \sec t,\ x = t \sin t$

Question 2

(a)Find the following limits: 4

(i) $\lim\limits_{x\to 1} \dfrac{\sqrt{x} - 1}{x^7 + 1}$ (ii) $\lim\limits_{x\to 0} \dfrac{\tan 3x}{4^x - 3^x}$ (iii) $\lim\limits_{x\to 0} \dfrac{\log(1 + 2x)}{x^3 + 2x}$ (iv) $\lim\limits_{x\to \infty} \dfrac{x^7 + 2x}{3 + x^7}$

(b)Write the Maclurin's series of the function: $f(x) = x \ln(1 + x^3)$. 2

(c)Show that : $\tanh^{-1} x = \frac{1}{2} \ln \frac{1+x}{1-x}$. 2

(d)Determine the extrema of : $f(x) = \frac{x}{3} + \frac{3}{x}$ $g(x) = x^{2/3}$ 3

Question 3 10

Integrate the following:

(a) $\int \dfrac{3x}{(4 + x^2)} dx$ (b) $\int \sqrt{1 - x^2}\, dx$ (c) $\int x\, e^x dx$

(d) $\int \cos^4 x \sin^3 x\, dx$ $(e) \int \tan^3 x\, dx$ (f) $\int \sec^4 \tan^3 x dx$

Question 4 10

(a)Find the area of the surface of revaluation generated by revolving about ***x***-axis the cycloid $x = a(t - \sin t),\ y = (1 - \cos t),\ 0 \le t \le 2\pi$

(b) Find the area bounded by the curves: $y = x^4,\ x = 2,\ x = 5,\ y = 0$.

(c) Find the volume generated by revolving, aboutx-axis, the area bounded by: $x = 2\cos\theta,\ y = 3\sin\theta$

Good Luck *Dr. Mohamed Eid* *Dr. Fathi Abdsallam*

Answer

Answer of Question 1

(a) $y` = 5x^4 + 2^{x^3} \ln 2 \,.\, 3x^2 - 1$

(b) $y` = \cosh x^3 \,.\, \text{sech}\, x \,.\, \tanh x + \sinh x^3 \,.\, 3x^2 . \text{sech}\, x$

(c) $y` = \cos x^2 \,.\, 2x + \frac{\sin x}{\cos x}$

(d) $y` = \frac{1}{1+x^2} + sec^2 x$

(e) $\sqrt{x} . \frac{y`}{2\sqrt{y}} + \frac{1}{2\sqrt{x}} \sqrt{y} = \frac{1}{\ln 10} \frac{1+y`}{x+y}$

Then $y` \left(\frac{1}{2} . \sqrt{\frac{x}{y}} - \frac{1}{\ln 10} \frac{1}{x+y} \right) = \frac{1}{\ln 10} \frac{1}{x+y} - \frac{1}{2} \sqrt{\frac{y}{x}}$

Then $y` = \frac{\frac{1}{\ln 10} \frac{1}{x+y} - \frac{1}{2} \sqrt{\frac{y}{x}}}{\frac{1}{2} . \sqrt{\frac{x}{y}} - \frac{1}{\ln 10} \frac{1}{x+y}}$

(f) $y` = \frac{\dot{y}}{\dot{x}} = \frac{\sec t + t \sec t . \tan t}{\sin t + t \cos t}$

----------*9-Marks*

Answer of Question 2

(a)(i) $\lim\limits_{x \to 0} \frac{\sqrt{x} - 1}{x^7 + 1} = \frac{1 - 1}{1 + 1} = 0$

(ii) $\lim\limits_{x \to 0} \frac{\tan 3x}{4^x - 3^x} = \frac{0}{0} = \lim\limits_{x \to 0} \frac{\frac{\tan 3x}{x}}{3^x \frac{((4/3)^x - 1)}{x}} = \frac{3}{\ln(4/3)}$

(iii) $\lim\limits_{x \to 0} \frac{\log(1 + 2x)}{x^3 + 2x} = \frac{0}{0} = \lim\limits_{x \to 0} \frac{\frac{\log(1 + 2x)}{x}}{x^2 + 2} = \frac{2/\ln 10}{2} = \frac{1}{\ln 10}$

(iv) $\lim\limits_{x \to \infty} \frac{x^7 + 2x}{3 + x^7} = \frac{\infty}{\infty} = \lim\limits_{x \to \infty} \frac{1 + \frac{2}{x^6}}{\frac{3}{x^7} + 1} = \frac{1 + 0}{0 + 1} = 1$

----------*4-Marks*

(b)Since $\ln x = x - \frac{x^2}{2} + \frac{x^3}{3} \ldots.$

Then $\ln(1 + x^3) = x^3 - \frac{x^6}{2} + \frac{x^9}{3} \ldots.$

Then $f(x) = x\ln(1 + x^3) = x^4 - \frac{x^7}{2} + \frac{x^{10}}{3}$ ....

----------*2-Marks*

(c)Proof

----------*2-Marks*

(d) $Since\ \ f`(x) = \frac{1}{3} - \frac{3}{x^2} = 0$. Then x = 3 , x = –3.

Then, the point (3, 2) is minimum because $f``(3) = \frac{6}{x^3} = \frac{2}{9}$

the point (–3, –2) is because maximum $f``(-3) = \frac{6}{x^3} = -\frac{2}{9}$

$Since\ \ g`(x) = \frac{2}{3}x^{-1/3} \neq 0\ \ and\ g`(0) is\ not\ real\ but\ g(0) = 0\ is\ real$

Using the first derivative test, we get

$g`(0^-) = g`(-1) = \frac{2}{3}\ \ and\ \ g`(0^+) = g`(1) = -\frac{2}{3}$

Then, the point (0, 0) is maximum.

----------*3-Marks*

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