Benha University
Faculty of Engineering- Shoubra
Industrial engineering Program
Final Examination

Subject: Mathematics 1
Code: EMP 101
Date: 31/12/2016
Duration : 2 hours

- **Answer all the following questions**
- **Illustrate your answers with sketches when necessary.**
- **The Exam. Consists of one Page**
- **No. of Questions: 2**
- **Total Mark:** 40 Marks

Question (1):

a) $f(x)=\begin{bmatrix} x^2+2 & x<0 \\ a\,x+b & 0\le x<1 \\ 3x+2x-x^2 & x\ge 1 \end{bmatrix}$, find a & b such that the function is continuous every where. [3]

b) Find the first derivative for the following functions

$$y = \ln\left[\frac{x^3+3x^2+5x}{x^5+7x^4}\right] + \cosh^{-1}(\cos x) + x^{\cos x}(\sin x)^{\tan x}$$ [9]

c) Evaluate the following limits i) $\lim_{x\to 1}\left[\frac{1}{\ln x}-\frac{1}{x-1}\right]$ ii) $\lim_{x\to\frac{\pi}{2}}[\sin x]^{\tan x}$ [6]

d) Expand using Taylor f(x) = x lnx about x = 1. [3]

Question (2)

a) Expand $(3-x^2)^5$ [3]

b) $A=\begin{pmatrix} 2 & -1 & 0 \\ 3 & 1 & 4 \\ -2 & 5 & 6 \end{pmatrix}$, find A^{-1} [4]

c) Resolve into partial fraction $\frac{x^4+3x^2+7x-1}{(x-1)(x+4)}$ [4]

d)Solve the system of equations $2x-y=-5$, $2y-x=13$ using Cramer and Gauss methods [8]

==

Dr. Eng. Khaled El Naggar

Model answer

Question 1

a) Since f(x) is continuous at x = 0 & at x = 1, therefore f(0) = $\lim\limits_{x\to 0^-}(x^2+2)=2$ = b,

also f(1) = $\lim\limits_{x\to 1^-}(a\,x+b)=a+b=4$. Hence b = 2.

b) Let y = u + v + w, $u=\ln\left[\dfrac{x^3+3x^2+5x}{x^5+7x^4}\right]=\ln(x^3+3x^2+5x)-\ln(x^5+7x^4)$, thus

$u` = \left(\dfrac{3x^2+6x+5}{x^3+3x^2+5x}\right)-\left(\dfrac{5x^4+28x^3}{x^5+7x^4}\right)$, and $v=\cosh^{-1}(\cos x)$, thus $v` = \dfrac{-\sin x}{\sqrt{\cos^2 x-1}}$.

$w = x^{\cos x}(\sin x)^{\tan x}$, hence ln w = cosx lnx + tanx ln(sinx) , thus

$w` = \dfrac{w`}{w}=-\sin x\ln x+\dfrac{\cos x}{x}+\sec^2 x\ln(\sin x)+1$

c-i) $\lim\limits_{x\to 1}\left[\dfrac{1}{\ln x}-\dfrac{1}{x-1}\right]=\lim\limits_{x\to 1}\left[\dfrac{x-1-\ln x}{(x-1)\ln x}\right]=\lim\limits_{x\to 1}\left[\dfrac{1-\dfrac{1}{x}}{\ln x+\dfrac{x-1}{x}}\right]$

$$=\lim_{x\to 1}\left[\frac{x-1}{x\ln x+x-1}\right]=\lim_{x\to 1}\left[\frac{1}{\ln x+2}\right]=\frac{1}{2}$$

ii) $\lim\limits_{x\to\frac{\pi}{2}}\tan x\ln[\sin x]=\lim\limits_{x\to\frac{\pi}{2}}\dfrac{\ln[\sin x]}{\cot x}=\lim\limits_{x\to\frac{\pi}{2}}\dfrac{\cos x/\sin x}{-\csc^2 x}=-\lim\limits_{x\to\frac{\pi}{2}}\cos x\sin x=0$, hence

$\lim\limits_{x\to\frac{\pi}{2}}[\sin x]^{\tan x}=1$.

d) Let g(x) = lnx, thus $g`(x)=\dfrac{1}{x}$, $g``(x)=-\dfrac{1}{x^2}$, $g```(x)=\dfrac{2}{x^3}$

Hence lnx = (x-1) $-\dfrac{(x-1)^2}{2}+\dfrac{(x-1)^3}{3}+\ldots$, therefore

x lnx = x (x-1) $-\dfrac{x\,(x-1)^2}{2}+\dfrac{x\,(x-1)^3}{3}+\ldots$

Answer of Question 2

a) $(3 - x^2)^5 = 3^5 + 5\,(3^4)(-x^2) + 10(3^3)(-x^2)^2 + 10(3^2)(-x^2)^3 + 5(3)(-x^2)^4 + (-x^2)^5$

b) $A^{-1} = \frac{1}{|A|}\text{adjoint}, \quad |A| = -2 \quad \text{and} \quad \text{adj}(A) = \begin{pmatrix} -14 & -26 & 17 \\ 6 & 12 & -8 \\ -4 & -8 & 5 \end{pmatrix}$

c) $\frac{x^4 + 3x^2 + 7x - 1}{(x-1)(x+4)} = x^2 - 3x + 16 + \frac{63 - 53x}{(x-1)(x+4)} == x^2 - 3x + 16 + \frac{2}{(x-1)} - \frac{55}{(x+4)}$

d) $\left(\begin{array}{cc|c} 2 & -1 & -5 \\ -1 & 2 & 13 \end{array}\right) \sim \left(\begin{array}{cc|c} 2 & -1 & -5 \\ 0 & 3 & 21 \end{array}\right) \sim \left(\begin{array}{cc|c} 6 & 0 & 6 \\ 0 & 3 & 21 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 7 \end{array}\right)$, therefore x = 1 and y = 7