Benha University
Faculty of Engineering- Shoubra
Electrical Engineering Department
2nd Year electrical power تخلفات

Final Term Exam
Date:24 th of December 2016
Mathematics 3A EMP 271
Duration : 3 hours

- Answer all the following question
- Illustrate your answers with sketches when necessary.
- The exam. Consists of one page
- No. of questions:6
- Total Mark: 80 Marks

1- A fair coin is tossed 3 times, X is the N^{o} of heads that come up on the first 2 tosses and Y is the N^{o} of heads that come up on tosses 2, 3. Construct the joint distribution and find marginal of X and Y, also find the variance of X , Y & P [(X + Y) >1, X > Y]. [12]

2- If $f(X) = c\, x^2 e^{-2x}$ is P.d.f., $x > 0$, find the mean and variance. [8]

3- A urn contains 30 red balls and 20 black balls, sample of 5 balls is selected at random. Let X is the number of red balls, find P(X=3), E(X), Var(X). [10]

4- Let X be a random variable with gamma distribution with alpha = 2, beta =1/5. Find the probability P(X > 30), E(X) and Var(X). [10]

5- Expand in fourier series the following periodic functions: [20]

$f(x) = x|x|$, $-1 < x < 1$,

$f(x) = x^2$, $0 < x < 2$, then deduce $\sum_{n=1}^{\infty} \frac{1}{n^4}$

6- Find Fourier transform and Fourier integral $f(x) = \begin{cases} 1 - x^2 & |x| < 1 \\ 0 & |x| > 1 \end{cases}$ [20]

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Model answer

1-

Y \ X	0	1	2	$f_1(x)$
0	1/8	1/8	0	2/8
1	1/8	2/8	1/8	4/8
2	0	1/8	1/8	2/8
$f_2(y)$	2/8	4/8	2/8	1

$E(X) = 0(2/8) + 1(4/8) + 2(2/8) = 1$, $E(Y) = 0(2/8) + 1(4/8) + 2(2/8) = 1$, $E(X^2) = 1(4/8) + 4(2/8) = 3/2$, $E(Y^2) = 1(4/8) + 4(2/8) = 3/2$, $Var(X) = Var(Y) = 1/2$.

$P[(X + Y) > 1, X > Y] = P(2,0) + P(2,1) = 1/8$

2- Since $f(X) = 4x^2e^{-2x}$ is gamma distribution with $\alpha = 3$ & $\beta = 2$, therefore $E(X) = \alpha / \beta = 3/2$ and variance is $\alpha / \beta^2 = 3/4$ and M.G.f $= \dfrac{\beta^\alpha}{(\beta - t)^\alpha} = \dfrac{2^3}{(2-t)^3}$

3- $N = 50$, $k = 30$, $n = 5$, therefore by using hypergeometric distribution

$$P(X=3) = [\,{}^{30}C_3\,]\,[\,{}^{20}C_2]\,/\,[{}^{50}C_5\,]$$

$E(X) = n\,(k/N) = 5(30/50)$ and $V(X) = (\frac{N-n}{N-1})n(\frac{k}{N})(1-\frac{k}{N}) = 5\,(\frac{45}{49})\,(\frac{30}{50})(\frac{20}{50})$

4- $P(X > 30) = \dfrac{1}{25}\int_{30}^{\infty} x\,e^{-x/5}\,dx$, put $y = x-30$, therefore

$$P(X > 30) = \frac{1}{25}\int_0^\infty (y+30)\,e^{-(y+30)/5}\,dy = \frac{e^{-6}}{25}\int_0^\infty y\,e^{-y/5}\,dy + \frac{6e^{-6}}{5}\int_0^\infty e^{-y/5}\,dy$$

Put $y/5 = z \Rightarrow dz = dy/5$, therefore

$$P(X > 30) = e^{-6}\int_0^\infty z\,e^{-z}\,dz + 6e^{-6}\int_0^\infty e^{-z}\,dz = 7e^{-6}$$

$E(X) = \alpha / \beta = 10$, $Var(X) = 50$

5- The functions f(x) = x|x| is even, therefore $a_0 = a_n = 0$, T = 1, therefore

$$b_n = \frac{2}{1}\int_0^1 x^2 \sin(\frac{n\pi x}{1})\,dx = [x^2(-\frac{1}{n\pi}\cos(\frac{n\pi x}{1})) - 2x(-\frac{1}{n^2\pi^2}\sin(\frac{n\pi x}{1})) + 2(\frac{1}{n^3\pi^3}\cos(\frac{n\pi x}{1}))]_0^1$$

$$= [(-\frac{1}{n\pi}\cos(\frac{n\pi}{1})) + 2(\frac{1}{n^3\pi^3}\cos(\frac{n\pi}{1}) - \frac{1}{n^3\pi^3})], \text{ thus } f(x) = \sum_{n=1}^{\infty} b_n \sin(\frac{n\pi x}{T}) = \sum_{n=1}^{\infty} b_n \sin(n\pi x)$$

This function is $f(x) = x^2$ neither even nor odd, therefore

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(\frac{n\pi x}{T}) + b_n \sin(\frac{n\pi x}{T}) \quad , T=1$$

$$a_0 = \frac{1}{1}\int_0^2 f(x)\,dx == \frac{1}{1}\int_0^2 (x^2)dx = (\frac{x^3}{3})\Big|_0^2 = \frac{8}{3}$$

$$a_n = \frac{1}{1}\int_0^2 (x^2)\cos(\frac{n\pi x}{1})\,dx = [(x^2)(\frac{1}{n\pi}\sin(\frac{n\pi x}{1})) - (2x)(-\frac{1}{n^2\pi^2}\cos(\frac{n\pi x}{1}))$$

$$+ 2(-\frac{1}{n^3\pi^3}\sin(\frac{n\pi x}{1}))]\Big|_0^2 = \frac{4}{n^2\pi^2}$$

$$b_n = \frac{1}{1}\int_0^2 (x^2)\sin(\frac{n\pi x}{1})\,dx = [(x^2)(\frac{-1}{n\pi}\cos(\frac{n\pi x}{1})) - (2x)(-\frac{1}{n^2\pi^2}\sin(\frac{n\pi x}{1}))$$

$$+ 2(\frac{1}{n^3\pi^3}\cos(\frac{n\pi x}{1}))]\Big|_0^2 = \frac{-4}{n\pi}$$

6- Since this function is even, therefore there is only Fourier Cosine transform such

that $F_c(\alpha) = \sqrt{2/\pi}\int_0^{\infty} f(x)\cos\alpha x\,dx = \sqrt{2/\pi}\int_0^{1} (1-x^2)\cos\alpha x\,dx$

$$= \sqrt{2/\pi}[(1-x^2)(\frac{\sin\alpha x}{\alpha}) - (-2x)(\frac{-\cos\alpha x}{\alpha^2}) + (-2)(\frac{-\sin\alpha x}{\alpha^3})]_0^1$$

$$= \sqrt{2/\pi}[\frac{-2\cos\alpha}{\alpha^2} + \frac{2\sin\alpha}{\alpha^3}] = \frac{4}{\sqrt{2\pi}}[\frac{\sin\alpha - \alpha\cos\alpha}{\alpha^3}]$$

Therefore Fourier integral f(x) is expressed by $f(x) = \sqrt{\frac{2}{\pi}}\int_0^{\infty} F_c(\alpha)\cos\alpha x\,d\alpha$