Benha University
Faculty of Engineering- Shoubra
Electrical Engineering Department
2nd Year electrical communication
تخلفات

Final Term Exam
Date:24-12- 2016
Mathematics 3A EMP 281
Duration : 3 hours

- Answer all the following question
- Illustrate your answers with sketches when necessary.
- The exam. Consists of one page
- No. of questions:7
- Total Mark: 80 Marks

1- Susan goes to work by one of two routes A or B. The prob. of going by route A is 30%. If she goes by route A, the prob. of being late is 5% and if she goes by route B, the prob. of being late is 10%. Given Susan is late for shool, find prob. that she went via route B. [10]

2- A box contains 7 blue , 8 white and 9 red balls, two balls are drawn without replacement. Let X is the number of blue balls and Y is the number of red balls, Find joint probability function, Cov(x,y), P(X+Y=2). [10]

3- $f(x,y) = cxy$, $0 < y < x < 1$. Find Cov(x,y) , $P[(X+Y)<1/2]$ [10]

4-Evaluate m.g.f. for the random variable of exponential and gamma distributions, then deduce μ'_r , r = 0,1,2 [10]

5- Expand the function $f(x) = x^2$, $0 \le x \le 2$, T = 4 in even sine harmonic. [10]

6- Expand in fourier series the following periodic functions: [20]

i) $\mathbf{f(x) = 10 - x}$, $\mathbf{5 < x < 15}$, ii) $\mathbf{f(x) = |\cos x|}$ $\mathbf{0 < x < 2\pi}$

7- Solve the integral equation $\int_0^\infty f(x)\sin(\alpha x)\,dx = \begin{cases} 1 & 0 \le \alpha < 1 \\ 2 & 1 \le \alpha < 2 \\ 0 & 2 \le \alpha \end{cases}$ [10]

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Model answer

1) P(A) = 0.3, P(B) = 0.7, P(L/A) = 0.05, P(L/B) = 0.1, P(B/L) = [P(L/B)P(B)]/P(L), where L: is Late event, P(L) = P(L/A)P(A) + P(L/B)P(B) = 0.05(0.3) + 0.1(0.7) = 0.085, so P(B/L) = 0.1(0.7)/0.085 = 0.824

2)

X \ Y	0	1	2	$f_1(x)$
0	P(WW) = 0.1014	2P(BW) = 0.2029	P(BB) = 0.0761	0.3804
1	2P(RW) = 0.2609	2P(BR) = 0.2283	0	0.4892
2	P(RR) = 0.1304	0	0	0.1304
$f_1(x)$	0.4927	0.4312	0.0761	1

P(X+Y=2) = f(1,1) + f(2,0) + f(0,2) = 0.2283 + 0.0761 + 0.1304 = 0.4348

E(Y) = 0(0.3804) + 1(0.4892) + 2(0.1304) = 0.75, E(X) = 0(0.4927) + 1(0.4312) + 2(0.0761) = 0.5834, E(XY) = 0(0.1014) + 0(0.2029) + 0(0.0761) + 0(0.2609) + 1(0.2283) + 2(0) + 0(0.1304) + 2(0) + 4(0) = 0.2283, therefore Cov(X,Y) = E(XY) - E(X) E(Y) = -0.2093

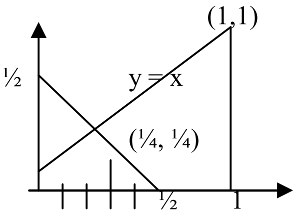
3) P First we have to get c, such that $\int_{y=0}^{1}\int_{x=y}^{1} cxy\,dx\,dy = 1 \Rightarrow \int_{y=0}^{1} \frac{y - y^3}{2}\,dy = 1 \Rightarrow c = 8$

The marginal probabilities $f_1(x)$, $f_2(y)$ are expressed by:

$f_1(x) = \int_0^x 8xy\,dy = 4xy^2\Big|_0^x = 4x^3$ and $f_2(y) = \int_y^1 8xy\,dx = 4yx^2\Big|_y^1 = 4(y - y^3)$

$E(X) = \int_0^1 x f_1(x)\,dx = \int_0^1 4x^4 dx = 4/5$, $E(Y) = \int_0^1 y f_2(y)\,dy = \int_0^1 4y(y - y^3)dy = 8/15$

$E(XY) = \int_{y=0}^{1}\int_{x=y}^{1} 8x^2y^2\,dx\,dy = 4/9$, Cov(X,Y) = E(XY) – E(X) E(Y) = 4/9 – (4/5)(8/15) = 0.0177



$P(X+Y < ½) = \int_{y=0}^{1/4}\int_{x=y}^{x=1/2-y} 8xy\,dx\,dy = \int_{y=0}^{1} 4x^2y\Big|_{x=y}^{1/2-y} dy = \int_{y=0}^{1} (1-4y)y\,dy = 5/6$

4) The moment generating function of a exponential distribution is expressed by $E(e^{tx}) = \int_0^{\infty} e^{tx}(\lambda\, e^{-\lambda x})dx = \int_0^{\infty} \lambda\, e^{-(\lambda-t)x}\, dx = \frac{\lambda}{(\lambda - t)}$, $\mu'_0 = 1, \mu'_1 = E(X) = \frac{1}{\lambda}$, $\mu'_2 = E(X^2) = \frac{2}{\lambda^2}$

The moment generating function of gamma distribution can be expressed by

$$E(e^{tx}) = \int_0^{\infty} e^{tx}\left(\frac{\beta^{\alpha}}{\Gamma\alpha} x^{\alpha-1}e^{-\beta x}\right)dx = \frac{\beta^{\alpha}}{\Gamma\alpha}\int_0^{\infty} x^{\alpha-1}e^{-(\beta-t)x}dx$$

Put $(\beta - t)x = y \Rightarrow dx = \frac{dy}{\beta - t}$, thus $E(e^{tx}) = \frac{\beta^{\alpha}}{(\beta-t)^{\alpha}\Gamma\alpha}\int_0^{\infty} y^{\alpha-1}e^{-y}dy = \frac{\beta^{\alpha}}{(\beta-t)^{\alpha}}$

5) $f(x) = \sum_{n=1}^{\infty} b_{2n} \sin\left(\frac{2n\pi x}{T}\right)$, where $a_0 = a_{2n} = 0$, and

$$b_{2n} = \frac{4}{T}\int_0^{T/2} f(x)\sin\left(\frac{2n\pi x}{T}\right) dx = \frac{4}{4}\int_0^{2} x^2 \sin\left(\frac{n\pi x}{2}\right) dx$$

$$= \left(x^2\left(-\frac{2\cos(\frac{n\pi x}{2})}{n\pi}\right) - 2x\left(-\frac{4\sin(\frac{n\pi x}{2})}{n^2\pi^2}\right) + 2\left(\frac{8\cos(\frac{n\pi x}{2})}{n^3\pi^3}\right)\right)\Bigg|_0^2$$

$$= \frac{-8}{n\pi}\cos n\pi + \frac{16(\cos(n\pi)-1)}{n^3\pi^3}$$

6- put X = x-10, then the function becomes f(X) = -X which is odd, -5 < X < 5, thus $a_0 = a_n = 0$

$$b_n = \frac{2}{5}\int_0^{5} -X\sin\left(\frac{n\pi}{5}\right)X\, dX = -\frac{2}{5}\left[X\left(-\cos\left(\frac{n\pi}{5}\right)X\right)\frac{5}{n\pi} - \left(-\sin\left(\frac{n\pi}{5}\right)X\right)\frac{25}{n^2\pi^2}\right]\Bigg|_0^5 = \frac{10}{n\pi}\cos(n\pi)$$, therefore

$$f(x) = \sum_{n=1}^{\infty} \frac{10\cos(n\pi)}{n\pi} \sin\left(\frac{n\pi}{5}\right)(x-10).$$

6-ii) This function is even cosine harmonic, therefore $a_0 = \frac{4}{T}\int_0^{T/2} f(x)\, dx =$

$$\frac{4}{\pi}\int_0^{\pi/2} \cos(x)\, dx = \frac{4}{\pi}$$

$$a_{2n} = \frac{4}{T}\int_0^{T/2} f(x)\cos\left(\frac{2n\pi x}{T}\right) dx = \frac{4}{\pi}\int_0^{\pi/2} \cos(x)\cos(2nx)\, dx = \frac{4\cos(n\pi)}{\pi(2n-1)(2n+1)}, \quad b_{2n} = 0$$

3-b) Since $T=\pi$, therefore

$$c_n = \frac{1}{2\pi}\int_{-T}^{T} f(x)\, e^{-i(\frac{n\pi x}{T})}\,dx = \frac{1}{2\pi}\int_{0}^{\pi} e^{-i(nx)}\,dx = \frac{i}{2\pi n}[e^{-i(n\pi)} - 1] = \frac{i}{2\pi n}[\cos(n\pi) - 1]$$

Thus $c_{2n-1} = \frac{-i}{\pi n}$, therefore $f(x) = \sum_{n=-\infty}^{\infty} c_{2n-1}\, e^{-i(2n-1)x}$,

4-a) $$F_S(\alpha) = \sqrt{2/\pi}\int_0^\infty \sin\alpha x\, dx = \sqrt{2/\pi}\begin{cases} 1 & 0 \le \alpha < 1 \\ 2 & 1 \le \alpha < 2 \\ 0 & 2 \le \alpha \end{cases}$$, therefore

$$f(x) = \sqrt{\frac{2}{\pi}}\int_0^\infty F_S(\alpha)\sin\alpha x\, d\alpha = \frac{2}{\pi}\Big[\int_0^1 \sin\alpha x\, d\alpha + \int_1^2 2\sin\alpha x\, d\alpha\Big] = \frac{2}{\pi}\Big[\frac{1+\cos\alpha - 2\cos 2\alpha}{x}\Big]$$

4b- i) we have to extend this function to be even such that:

$$a_0 = \frac{2}{1}\int_0^1 x\, dx = \left(\frac{2x^2}{2}\right)_0^1 = 1$$

$$a_n = \frac{2}{1}\int_0^1 x\, \cos\left(\frac{n\pi x}{1}\right)dx = 2\Big[x\,\frac{\sin(n\pi x)}{n\pi} + \frac{\cos(n\pi x)}{n^2\pi^2}\Big]_0^1 = 2\Big[\frac{\cos(n\pi) - 1}{n^2\pi^2}\Big]$$

Therefore $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{T}\right) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{-4}{n^2\pi^2}\cos(2n\pi x)$,

4b-ii) Thus $f(x) = \sum_{n=1}^{\infty} a_{2n-1}\cos(2n-1)x + \sum_{n=1}^{\infty} b_{2n-1}\sin(2n-1)x$

$$a_{2n-1} = \frac{2}{1}\int_0^1 x\cos(2n-1)\pi x\, dx$$

$$= \frac{2}{1}\left(x\left(\frac{\sin(2n-1)\pi x}{(2n-1)\pi}\right) - \left(\frac{-\cos(2n-1)\pi x}{(2n-1)^2\pi^2}\right)\right)_0^1 = \frac{-4}{(2n-1)^2\pi^2}$$

$$b_{2n-1} = \frac{2}{1}\int_0^1 x\ \sin(2n-1)\pi x\, dx$$

$$= \frac{2}{1}\left(x\left(\frac{-\cos(2n-1)\pi x}{(2n-1)\pi}\right) - \left(\frac{-\sin(2n-1)\pi x}{(2n-1)^2\pi^2}\right)\right)_0^1$$

$$= \frac{2}{\pi(2n-1)}$$

Therefore $f(x) = \sum_{n=1}^{\infty} b_{2n-1} \sin(2n-1)x = -\frac{4}{\pi}\sum_{n=1}^{\infty} \frac{\cos n\pi}{(2n-1)^2} \sin(2n-1)x$