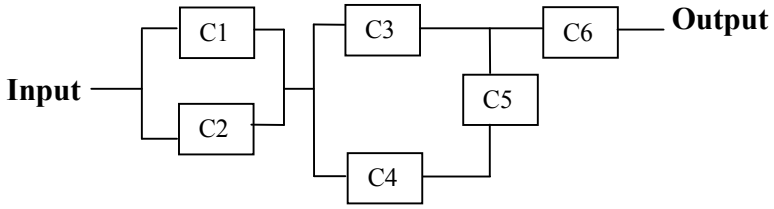
Benha University Faculty of Engineering- Shoubra (**Electrical**)Engineering Department		Final term exam Date: **9-1-2017** Mathematics **3A** Code: **EMP 271** Duration : **3 hours**
Answer the following questions	No. of questions : **4**	Total Mark: 80

1-a) Given the system of components c_i, i = 1,2,…,6 with reliabilities 0.8, 0.7, 0.9, 0.6, 0.7, 0.8 respectively. Calculate all possible overall reliability if one of the components is out of order.



1-b) The joint density function of two random variables X & Y is given by $f(x,y) = \frac{xy}{96}$, $0 < x < 4$, $1 < y < 5$. Find Cov(x,y) and check for independence, then find P(X+Y > 3/2) and cov(x,y).

2-a) Four sided die is rolled twice, let r.v. X is difference of the two scores and r.v. Y is the sum of the two scores. Discuss the joint distribution.

2- b) A coin is biased so that heads is twice the tails for three independent tosses of the coin. Find the probability of getting at most two heads, and standard deviation.

3-a) A card is drawn at random from a standard deck of playing cards. What is the Prob. that the card is less than a 7 given: A) The card is not a 2, B) The card is a heart, C) The card is a 3 or 4

3-b) Find complex Fourier for $f(x) = e^{-x}$, $-2 < x < 2$

4-a) Find Fourier transform and Fourier integral for $f(x) = \begin{cases} 1 & 0 < x < a \\ -1 & -a < x < 0 \\ 0 & |x| > a \end{cases}$

4-b) Suppose that X is a normal random variable with mean 5. If $P\{X>9\} = 0.2$, approximately. What is var(X)?

Dr. eng. Khaled El Naggar

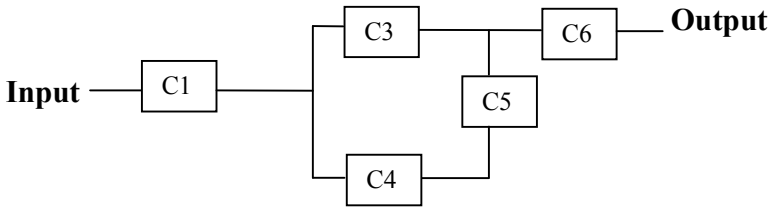
Tables of the Normal Distribution

Probability Content from -oo to Z

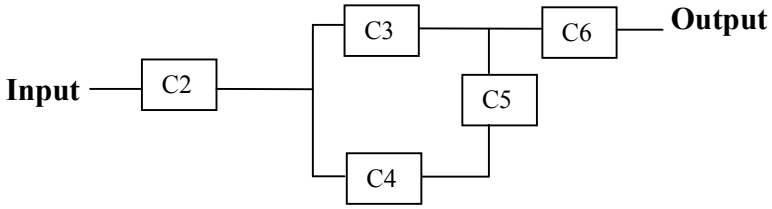
Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

Model answer

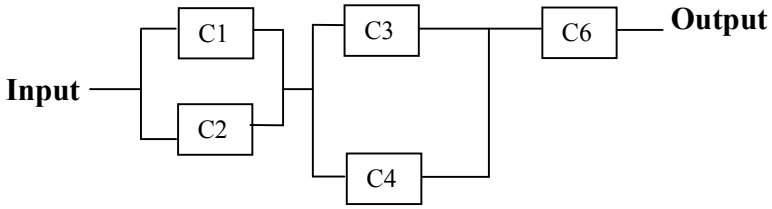
Answer of Question 1a



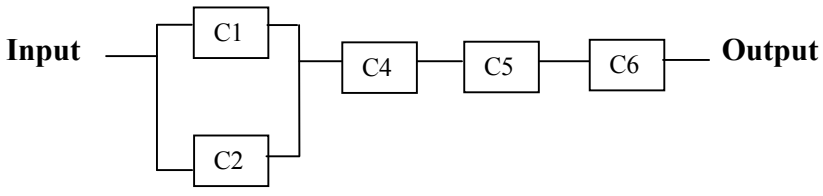
R = c1 . [1-(1-c3)(1- c4. c5)] . c6



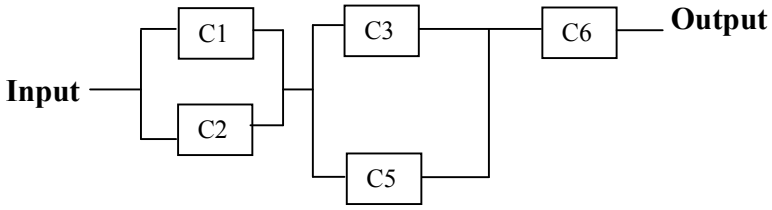
R = c2. [1-(1- c3)(1- c4. c5)] . c6



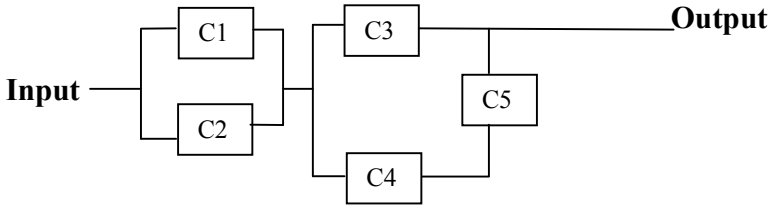
R = [1-(1- c1)(1- c2)][1-(1-c3)(1-c4)] . c6



R = [1-(1-c1)(1-c2)] c4. c5. c6



$R = [1- (1- c1)(1- c2)][1-(1- c3)(1- c5)] \,.\, c6$



$R = [1- (1- c1)(1- c2)][1-(1- c3)(1- c5)]$

Answer of Question 1b

The marginal probabilities $f_1(x)$, $f_2(y)$ are expressed by:

$f_1(x) = \int_1^5 \frac{xy}{96}\,dy = \frac{xy^2}{192}\Big|_1^5 = \frac{x}{8}$ and $f_2(y) = \int_0^4 \frac{xy}{96}\,dx = \frac{x^2y}{192}\Big|_0^4 = \frac{y}{12}$, therefore they are independent and $E(X) = \int_0^4 \frac{x^2}{8}\,dx = \frac{x^3}{24}\Big|_0^4 = \frac{8}{3}$, $E(Y) = \int_1^5 \frac{y^2}{12}\,dx = \frac{y^3}{36}\Big|_1^5 = \frac{31}{9}$, but $E(XY) = E(X)E(Y) = \frac{248}{27}$ and $E(2X + 3Y) = 2E(X) + 3E(Y) = \frac{16}{3} + \frac{31}{3} = \frac{47}{3}$, thus

cov (x, y) = 0

Answer of Question 2a

x / y	0	1	2	3	f_y
2	1/16	0	0	0	1/16
3	0	2/16	0	0	2/16
4	1/16	0	2/16	0	3/16
5	0	2/16	0	2/16	4/16
6	1/16	0	2/16	0	3/16
7	0	2/16	0	0	2/16
8	1/16	0	0	0	1/16
f_x	1/4	6/16	4/16	2/16	1

Answer of Question 2b

P(H) = 2 P(T), therefore P(H) = 2/3 = P, and $P(H \leq 2) = \sum_{x=0}^{2} {}^{3}c_{x} (2/3)^{x} (1/3)^{3-x}$

$P(x=0) = {}^{3}c_{0} (2/3)^{0} (1/3)^{3}$, $P(x = 1) = \sum_{x=0}^{1} {}^{3}c_{x} (2/3)^{x} (1/3)^{3-x}$, n = 3

$P(x =2) = \sum_{x=0}^{2} {}^{3}c_{x} (2/3)^{x} (1/3)^{3-x}$, var(x) = npq = 3(2/3)(1/3) = 2/3, therefore standard deviation = $\sqrt{\frac{2}{3}}$.

Answer of Question 3a

D = {card is less than a 7} = {24 cards}

A = {card is not a 2} = {48 cards}, therefore P(D/A) = 20/48 = 5/12

B = {card is a heart} = {13 cards}, therefore P(D/B) = 6/13,

C = {card is a 3 or 4} = {8 cards}, therefore P(D/C) = 1.

Answer of Question 3b

Since T = 2, therefore

$$c_n = \frac{1}{2T}\int_{-T}^{T} f(x)\, e^{-i\left(\frac{n\pi x}{T}\right)} dx = \frac{1}{4}\int_{-2}^{2} e^{-x}\, e^{-i\left(\frac{n\pi x}{2}\right)} dx = \frac{1}{4}\int_{-2}^{2} e^{-\left(1+\frac{in\pi}{2}\right)x} dx$$

$$= -\frac{1}{2(2+in\pi)}\left(e^{-\left(1+\frac{in\pi}{2}\right)x}\right)_{-2}^{2} = \frac{1}{2(2+in\pi)}\left(e^{(2+in\pi)} - e^{-(2+in\pi)}\right) = \frac{i}{(2+in\pi)}\sin(2+in\pi)$$

$$= -\frac{1}{(2+in\pi)}(\cos 2 \sinh n\pi - i \sin 2 \cosh n\pi)$$

Answer of Question 4a

Since the function f(x) is odd therefore

$$f(x) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} F_S(\alpha) \sin \alpha x \, d\alpha$$

$F_S(\alpha) = \sqrt{2/\pi} \int_0^{a} \sin\alpha x \, dx = \sqrt{2/\pi}\left(\frac{-\cos \alpha x}{\alpha}\right)_0^{a} = \sqrt{2/\pi}\left(\frac{1-\cos \alpha a}{\alpha}\right)$ Therefore

$f(x) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} F_S(\alpha) \sin \alpha x \, d\alpha = \sqrt{\frac{2}{\pi}} \int_0^{\infty} \sqrt{2/\pi}\left(\frac{1-\cos \alpha a}{\alpha}\right) \sin \alpha x \, d\alpha$, hence

$$f(x) = \frac{2}{\pi} \int_0^{\infty} \left(\frac{1-\cos \alpha a}{\alpha}\right) \sin \alpha x \, d\alpha = \begin{cases} 1 & 0 < x < a \\ -1 & -a < x < 0 \\ 0 & |x| > a \end{cases}.$$

At x = 0, $f(x) = \frac{2}{\pi} \int_0^{\infty} \left(\frac{1-\cos \alpha a}{\alpha}\right) \sin \alpha x \, d\alpha = 0 = ½ [f(0^+) + f(0^-)] = ½[1-1]$, so Fourier integral is verified.

Answer of Question 4b

The probability function is an exponential distribution with c = 1/40

$P(X > x) = 1 - P(X<x) = 1 - \int_0^{x} \frac{1}{40} e^{-x/40} dx = e^{-x/40}$. To get the median a such that $P(X< a) = 0.5$, therefore $\int_0^{a} \frac{1}{40} e^{-x/40} dx = 0.5$, thus $1 - e^{-a/40} = 0.5 \Rightarrow a = -40 \ln(0.5) = 27.726$, mean = 40, Variance = 1600 and so standard deviation = 40.

Intended Learning Outcomes of Course (ILOS)

a- Knowledge and Understanding

On completing this course, students will be able to:

a.1) Recognize concepts and theories of mathematics and sciences, appropriate to the discipline. (a.1)

a.2) Recognize methodologies of solving engineering problems. (a.5)

b- Intellectual Skills

At the end of this course, the students will be able to:

b.1) Select appropriate mathematical and computer-based methods for modeling and analyzing problems. (b.1)

b.2) Select appropriate solutions for engineering problems based on analytical thinking. (b.2)

b.3) Solve engineering problems, often on the basis of limited and possibly contradicting information. (b.7)

c- Professional Skills

On completing this course, the students are expected to be able to:

c.1) Apply knowledge of mathematics, science, information technology, design, business context and engineering practice to solve engineering problems. (c.1)

c.2) Apply numerical modeling methods to engineering problems. (c.7)

Question	Marks	Achieved ILOS
1	20	a1,b1
2	20	a1,c1
3	20	a5, b2,b7
4	20	c7,b7