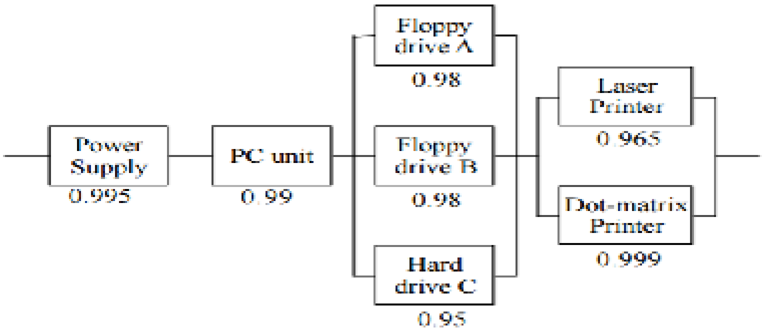
Benha University Faculty of Engineering- Shoubra (**Electrical**)Engineering Department		Final term exam Date: **12-1-2017** Mathematics **3A** Code: **EMP 281** Duration : **3 hours**
Answer the following questions	No. of questions : **4**	Total Mark: 80

1- a) Find Reliability of the system



If the Dot-matrix printer is out of order, calculate the reliability.

1-b) If the Random variables X and Y are independent where the probability density function of X is $f_x = 6x(1-x)$, $0 < x < 1$ and the probability density function of Y is $f_y = 12y^2(1-y)$, $0 < y < 1$. Find the Joint probability f(x,y) and $P(X+Y < 1)$.

2-a) A random distribution of 3 balls into 3 cells, where r.v. X is number of balls in cell 1 and r.v. Y is the number of occupied cells. Discuss the joint distribution**.**

If r.v. Z is the number of balls in cell 2, discuss the joint distribution between X & Z.

2-b) Expand into complex Fourier series the periodic function $f(x) = \begin{cases} 0, & -\pi < x < 0 \\ 1, & 0 < x < \pi \end{cases}$ of period 2π

3-a) Find Fourier transform and Fourier integral $f(x) = \begin{cases} 1 - x^2 & |x| < 1 \\ 0 & |x| > 1 \end{cases}$

3-b) In a bolt factory, machines A,B,C manufacture such that machine A produce twice that of machine B which produce half that of machine C, 2%, 4%, 5% are defective bolts respectively , a bolt is drawn at random and it is a defective quality , what is the probability that it was produced by machine A, B, C.

4-a) When Justin is goal-keeper, Shaunie manages to score an average of once for every 10 shots he takes. If Shaunie takes 12 shots, find the following probability that he scores at most twice.

4-b) If $f(x) = c\,x^2 e^{-2x}$ is P.d.f., $x > 0$, find c, mean and variance.

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Model answer

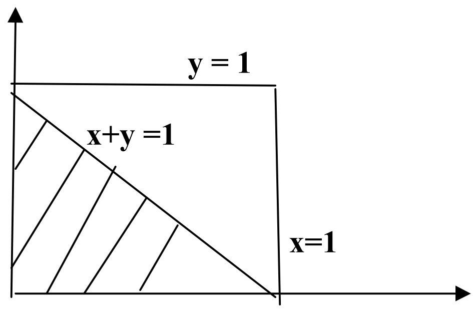
Answer of Question 1a

Reliability = 0.995(0.99)[1-(0.02)(0.02)(0.05)][1-(0.035)(0.001)] = 0.985.
Since the Dot-matrix printer is out of order, therefore the reliability will be
0.995(0.99)[1-(0.02)(0.02)(0.05)](0.965) = 0.951

Answer of Question 1b

Since X and Y are independent, therefore $f(x,y) = f_x\, f_y$

Hence prob. density function $f(x,y) = 72\, x\, y^2\, (1-x)(1-y)$, $0 < x < 1$ & $0 < y < 1$



$$P(X+Y < 1) = \int_0^1 \int_0^{1-y} 72\, x\, y^2\, (1-x)(1-y)\, dx$$

Answer of Question 2a

X \ Y	0	1	2	3	f_y
1	2/27	0	0	1/27	3/27
2	6/27	6/27	6/27	0	18/27
3	0	6/27	0	0	6/27
f_x	8/27	12/27	6/27	1/27	1

X \ Z	0	1	2	3	f_z
0	1/27	3/27	3/27	1/27	8/27
1	3/27	6/27	3/27	0	12/27
2	3/27	3/27	0	0	6/27
3	1/27	0	0	0	1/27
f_x	8/27	12/27	6/27	1/27	1

Answer of Question 2b

Since $T=\pi$, therefore

$$c_n = \frac{1}{2\pi}\int_{-T}^{T} f(x)\, e^{-i(\frac{n\pi x}{T})}dx = \frac{1}{2\pi}\int_{0}^{\pi} e^{-i(nx)}dx = \frac{i}{2\pi n}[e^{-i(n\pi)}-1] = \frac{i}{2\pi n}[\cos(n\pi)-$$

Thus $c_{2n-1} = \frac{-i}{\pi n}$, therefore $f(x) = \sum_{n=-\infty}^{\infty} c_{2n-1}\, e^{-i(2n-1)x}$,

Answer of Question 3a

Since this function is even, therefore there is only Fourier Cosine transform

such that $F_c(\alpha) = \sqrt{2/\pi}\int_0^{\infty} f(x)\cos\alpha x\, dx = \sqrt{2/\pi}\int_0^{1}(1-x^2)\cos\alpha x\, dx$

$$= \sqrt{2/\pi}\left[(1-x^2)\left(\frac{\sin\alpha x}{\alpha}\right)-(-2x)\left(\frac{-\cos\alpha x}{\alpha^2}\right)+(-2)\left(\frac{-\sin\alpha x}{\alpha^3}\right)\right]_0^1$$

$$= \sqrt{2/\pi}\left[\frac{-2\cos\alpha}{\alpha^2} + \frac{2\sin\alpha}{\alpha^3}\right] = \frac{4}{\sqrt{2\pi}}\left[\frac{\sin\alpha - \alpha\cos\alpha}{\alpha^3}\right]$$

Therefore Fourier integral f(x) is expressed by $f(x) = \sqrt{\frac{2}{\pi}} \int_0^\infty F_c(\alpha)\cos\alpha x \, d\alpha$

Answer of Question 3b

Let the defective event is D and the probability of machines A,B,C are 2/5, 1/5, 2/5 respectively, also P(D/A) = 0.02, P(D/B) = 0.04, P(D/C) = 0.05, therefore $P(C/D) = \frac{P(D/C)P(C)}{P(D)}$, $P(B/D) = \frac{P(D/B)P(B)}{P(D)}$, P(D) = P(D/A)P(A) + P(D/B)P(B) + P(D/C)P(C)

Answer of Question 4a

By using Binomial distribution, n = 12, p = 0.1, therefore

$$p(x \le 2) = \sum_{x=0}^{2} {}^{12}c_x (0.1)^x (0.9)^{12-x}$$

Answer of Question 4b

Since $f(X) = 4x^2e^{-2x}$ is gamma distribution with α = 3 & β = 2, therefore E(X) = α / β = 3/2 and variance is $\alpha / \beta^2 = 3/4$

- Intended Learning Outcomes of Course (ILOS)

a- Knowledge and Understanding

On completing this course, students will be able to:

a- 1 - Recognize concepts and theories of mathematics and sciences (a1)

a- 2 - Recognize methodologies of solving engineering problems, data collection interpretation. (a6)

b- Intellectual Skills

At the end of this course, the students will be able to:

b- 1 - Select appropriate mathematical and computer-based methods for modeling and analyzing problems. (b1)

b- 2 - Select appropriate solutions for engineering problems based on analytical thinking. (b3)

b- 3 - Solve engineering problems, often on the basis of limited and possibly contradicting information. (b8)

c- Professional Skills

On completing this course, the students are expected to be able to:

c- 1 - Apply knowledge of mathematics, science, information technology, design, busine

c- 2 - Apply numerical modeling methods to engineering problems. (c7)

d- General Skills

At the end of this course, the students will be able to:

d-1- Work in stressful environment and within constraints. (d2)

Questions	Total marks	Achieved ILOS
Q1	20	b1
Q2	20	a1
Q3	20	a2, c1
Q4	20	b2

Board of examiners: Dr. eng. Khaled El Naggar