Benha University
Faculty of Engineering (Shoubra)
Electrical Engineering Department
Postgraduate (master)

Final Term Exam (Model Answer)
Date: 12/01/2019
Advanced Studies in Electrical Power System Analysis
Duration: 3 Hours

- Answer all the following questions.
- Illustrate your answers with sketches when necessary.
- The exam consists of two pages.
- Number of questions: 4
- Total Mark: 60 Marks

**Q.1** [15 marks] Achieved ILOS a1, a2, a3, a4, a5, b1, b2, b4, b5, b7, b11, c1, d1, d2, d8

Fig.1 shows the one-line diagram of a simple three –bus power system with generators at buses 1 and 3. The magnitude of voltage at bus 1 is adjusted to 1.05 pu. Voltage magnitude at bus 3 is fixed at 1.04 pu with a real power generation of 200 MW. A load consisting of 400 MW and 250 MVAR is taken from bus 2. Line impedances are marked in per unit on a 100MVA base and the line charging susceptances are neglected. Obtain the power flow solution by the Gauss-Seidel method (two iterations only) including line flows and line losses.

(a) Line impedances are converted to admittances

$$y_{12} = \frac{1}{0.02 + j0.04} = 10 - j20$$

Similarly, $y_{13} = 10 - j30$ and $y_{23} = 16 - j32$. The admittances are marked on the network shown in Figure 6.10.

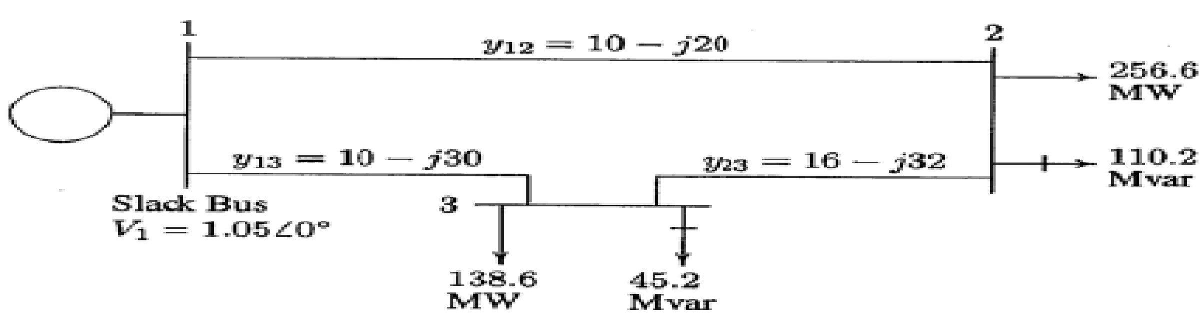
At the P-Q buses, the complex loads expressed in per units are

$$S_2^{sch} = -\frac{(256.6 + j110.2)}{100} = -2.566 - j1.102 \quad \text{pu}$$

$$S_3^{sch} = -\frac{(138.6 + j45.2)}{100} = -1.386 - j0.452 \quad \text{pu}$$

Since the actual admittances are readily available in Figure 6.10, for hand calculation, we use (6.28). Bus 1 is taken as reference bus (slack bus). Starting from an initial estimate of $V_2^{(0)} = 1.0 + j0.0$ and $V_3^{(0)} = 1.0 + j0.0$, $V_2$ and $V_3$ are computed from (6.28) as follows

$$V_2^{(1)} = \frac{\frac{P_2^{sch} - jQ_2^{sch}}{V_2^{*(0)}} + y_{12}V_1 + y_{23}V_3^{(0)}}{y_{12} + y_{23}}$$



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$$= \frac{\frac{-2.566+j1.102}{1.0-j0} + (10-j20)(1.05+j0) + (16-j32)(1.0+j0)}{(26-j52)}$$

$$= 0.9825 - j0.0310$$

and

$$V_3^{(1)} = \frac{\frac{P_3^{sch}-jQ_3^{sch}}{V_3^{*(0)}} + y_{13}V_1 + y_{23}V_2^{(1)}}{y_{13}+y_{23}}$$

$$= \frac{\frac{-1.386+j0.452}{1-j0} + (10-j30)(1.05+j0) + (16-j32)(0.9825-j0.0310)}{(26-j62)}$$

$$= 1.0011 - j0.0353$$

For the second iteration we have

$$V_2^{(2)} = \frac{\frac{-2.566+j1.102}{0.9825+j0.0310} + (10-j20)(1.05+j0) + (16-j32)(1.0011-j0.0353)}{(26-j52)}$$

$$= 0.9816 - j0.0520$$

and

$$V_3^{(2)} = \frac{\frac{-1.386+j0.452}{1.0011+j0.0353} + (10-j30)(1.05+j0) + (16-j32)(0.9816-j0.052)}{(26-j62)}$$

$$= 1.0008 - j0.0459$$

The process is continued and a solution is converged with an accuracy of $5 \times 10^{-5}$ per unit in seven iterations as given below.

$$V_2^{(3)} = 0.9808 - j0.0578 \qquad V_3^{(3)} = 1.0004 - j0.0488$$

$$V_2^{(4)} = 0.9803 - j0.0594 \qquad V_3^{(4)} = 1.0002 - j0.0497$$
$$V_2^{(5)} = 0.9801 - j0.0598 \qquad V_3^{(5)} = 1.0001 - j0.0499$$
$$V_2^{(6)} = 0.9801 - j0.0599 \qquad V_3^{(6)} = 1.0000 - j0.0500$$
$$V_2^{(7)} = 0.9800 - j0.0600 \qquad V_3^{(7)} = 1.0000 - j0.0500$$

The final solution is

$$V_2 = 0.9800 - j0.0600 = 0.98183\angle -3.5035^\circ \quad \text{pu}$$
$$V_3 = 1.0000 - j0.0500 = 1.00125\angle -2.8624^\circ \quad \text{pu}$$

(b) With the knowledge of all bus voltages, the slack bus power is obtained from (6.27)

$$\begin{aligned} P_1 - jQ_1 &= V_1^*[V_1(y_{12} + y_{13}) - (y_{12}V_2 + y_{13}V_3)] \\ &= 1.05[1.05(20 - j50) - (10 - j20)(0.98 - j.06) - \\ &\quad (10 - j30)(1.0 - j0.05)] \\ &= 4.095 - j1.890 \end{aligned}$$

or the slack bus real and reactive powers are $P_1 = 4.095$ pu $= 409.5$ MW and $Q_1 = 1.890$ pu $= 189$ Mvar.

(c) To find the line flows, first the line currents are computed. With line charging capacitors neglected, the line currents are

$$I_{12} = y_{12}(V_1 - V_2) = (10 - j20)[(1.05 + j0) - (0.98 - j0.06)] = 1.9 - j0.8$$
$$I_{21} = -I_{12} = -1.9 + j0.8$$
$$I_{13} = y_{13}(V_1 - V_3) = (10 - j30)[(1.05 + j0) - (1.0 - j0.05)] = 2.0 - j1.0$$
$$I_{31} = -I_{13} = -2.0 + j1.0$$
$$I_{23} = y_{23}(V_2 - V_3) = (16 - j32)[(0.98 - j0.06) - (1 - j0.05)] = -.64 + j.48$$
$$I_{32} = -I_{23} = 0.64 - j0.48$$

The line flows are

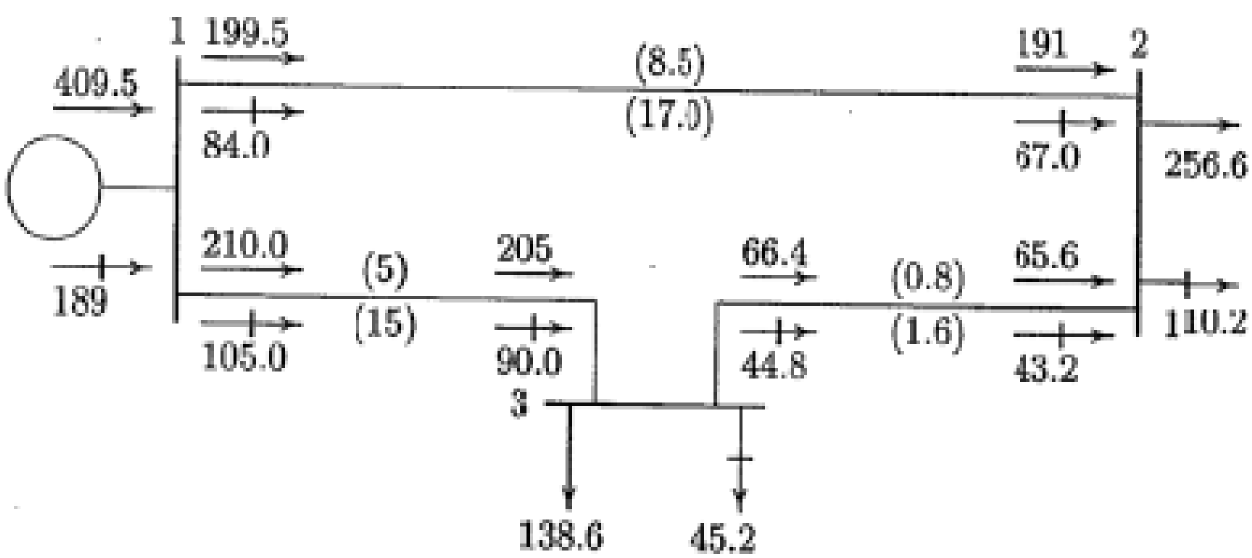
$$\begin{aligned} S_{12} = V_1 I_{12}^* &= (1.05 + j0.0)(1.9 + j0.8) = 1.995 + j0.84 \ \text{pu} \\ &= 199.5 \ \text{MW} + j84.0 \ \text{Mvar} \\ S_{21} = V_2 I_{21}^* &= (0.98 - j0.06)(-1.9 - j0.8) = -1.91 - j0.67 \ \text{pu} \\ &= -191.0 \ \text{MW} - j67.0 \ \text{Mvar} \\ S_{13} = V_1 I_{13}^* &= (1.05 + j0.0)(2.0 + j1.0) = 2.1 + j1.05 \ \text{pu} \\ &= 210.0 \ \text{MW} + j105.0 \ \text{Mvar} \end{aligned}$$

$$S_{31} = V_3 I_{31}^* = (1.0 - j0.05)(-2.0 - j1.0) = -2.05 - j0.90 \text{ pu}$$
$$= -205.0 \text{ MW} - j90.0 \text{ Mvar}$$
$$S_{23} = V_2 I_{23}^* = (0.98 - j0.06)(-0.656 + j0.48) = -0.656 - j0.432 \text{ pu}$$
$$= -65.6 \text{ MW} - j43.2 \text{ Mvar}$$
$$S_{32} = V_3 I_{32}^* = (1.0 - j0.05)(0.64 + j0.48) = 0.664 + j0.448 \text{ pu}$$
$$= 66.4 \text{ MW} + j44.8 \text{ Mvar}$$

and the line losses are

$$S_{L\,12} = S_{12} + S_{21} = 8.5 \text{ MW} + j17.0 \text{ Mvar}$$
$$S_{L\,13} = S_{13} + S_{31} = 5.0 \text{ MW} + j15.0 \text{ Mvar}$$
$$S_{L\,23} = S_{23} + S_{32} = 0.8 \text{ MW} + j1.60 \text{ Mvar}$$

The power flow diagram is shown in Figure 6.11, where real power direction is indicated by → and the reactive power direction is indicated by ↦. The values within parentheses are the real and reactive losses in the line.



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**Q.2** **[15 marks]** **Achieved ILOS a1, a3, a4, a5, b1, b2, b4, b5, b7, b11, c1, c3, d1, d8**

The one line diagram of a simple three-bus power system is shown in Fig.2. Each generator is represented by an emf behind the transient reactance. All impedances are expressed in per unit on a common 100 MVA base, and for simplicity, resistances are neglected. The following assumptions are made: (i) Shunt capacitances are neglected and the system is considered on no-load. (ii) All generators are running at their rated voltage and rated frequency with their emfs in phase. Determine the fault current, the bus voltages, and the line currents during the fault when a balanced three phase fault with fault impedance $Z_f$ = 0.16 per unit occurs on **Bus 3.**

To find Thévenin impedance viewed from the faulted bus (bus 3), we convert the delta formed by buses 123 to an equivalent Y as shown

$$Z_{1s} = \frac{(j0.125)(j0.15)}{j0.525} = j0.0357143$$

$$Z_{2s} = \frac{(j0.125)(j0.25)}{j0.525} = j0.0595238$$

$$Z_{3s} = \frac{(j0.15)(j0.25)}{j0.525} = j0.0714286$$

Combining the parallel branches, the positive-sequence Thévenin impedance is

$$Z_{33}^{1} = \frac{(j0.2857143)(j0.3095238)}{j0.5952381} + j0.0714286$$
$$= j0.1485714 + j0.0714286 = j0.22$$

This is shown in Figure 10.18(a).


(a) Positive-sequence network (b) Negative-sequence network

**Reduction of the positive sequence thevenin equivalent network**

Since the negative-sequence impedance of each element is the same as the positive-sequence impedance, we have

$$Z_{33}^{2} = Z_{33}^{1} = j0.22$$

To find Thévenin impedance viewed from the faulted bus (bus 3), we convert the delta formed by buses 123 to an equivalent Y as shown

$$Z_{1s} = \frac{(j0.30)(j0.35)}{j1.3625} = j0.0770642$$
$$Z_{2s} = \frac{(j0.30)(j0.7125)}{j1.3625} = j0.1568807$$
$$Z_{3s} = \frac{(j0.35)(j0.7125)}{j1.3625} = j0.1830257$$

Combining the parallel branches, the zero-sequence Thévenin impedance is

$$Z_{33}^{0} = \frac{(j0.4770642)(j0.2568807)}{j0.7339449} + j0.1830275$$
$$= j0.1669725 + j0.1830275 = j0.35$$



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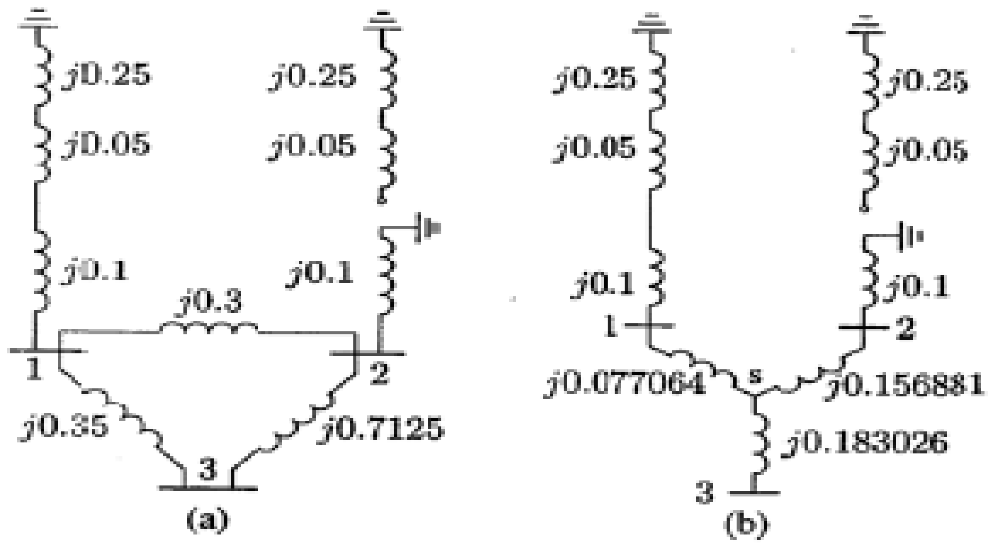
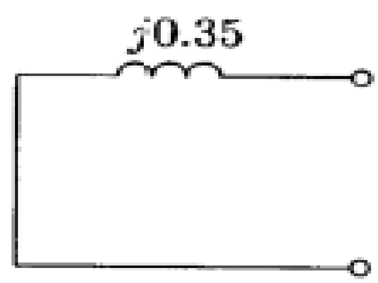



Zero sequence network diagram for problem (2)




Zero sequence network for problem (2)

(a) Balanced three-phase fault at bus 3.

Assuming the no-load generated emfs are equal to 1.0 per unit, the fault current is

$$I_3^a(F) = \frac{V_{3(0)}^a}{Z_{33}^1 + Z_f} = \frac{1.0}{j0.22 + j0.1} = -j3.125 \quad \text{pu}$$
$$= 820.1\angle -90^\circ \quad \text{A}$$

(b) Single line-to-ground fault at bus 3.

$$I_3^0 = I_3^1 = I_3^2 = \frac{V_3^a(0)}{Z_{33}^1 + Z_{33}^2 + Z_{33}^0 + 3Z_f} = \frac{1.0}{j0.22 + j0.22 + j0.35 + 3(j0.1)} = -j0.9174 \text{ pu}$$

The fault current is

$$\begin{bmatrix} I_3^a \\ I_3^b \\ I_3^c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_3^0 \\ I_3^0 \\ I_3^0 \end{bmatrix} = \begin{bmatrix} 3I_3^0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -j2.7523 \\ 0 \\ 0 \end{bmatrix} \text{ pu}$$

*c) A double line to ground fault at bus 3 through a fault impedance $Z_f$=j0.1 per unit.*

$$I_3^1 = \frac{V_{3(0)}^a}{Z_{33}^1 + \frac{Z_{33}^2(Z_{33}^0+3Z_f)}{Z_{33}^2+Z_{33}^0+3Z_f}} = \frac{1}{j0.22 + \frac{j0.22(j0.35+j0.3)}{j0.22+j0.35+j.3}} = -j2.6017 \text{ pu}$$

The negative-sequence component of current from (10.87) is

$$I_3^2 = -\frac{V_{3(0)}^a - Z_{33}^1 I_3^1}{Z_{33}^2} = -\frac{1-(j0.22)(-j2.6017)}{j0.22} = j1.9438 \text{ pu}$$

The zero-sequence component of current from (10.86) is

$$I_3^0 = -\frac{V_{3(0)}^a - Z_{33}^1 I_3^1}{Z_{33}^0 + 3Z_f} = -\frac{1-(j0.22)(-j2.6017)}{j0.35+j0.3} = j0.6579 \text{ pu}$$

and the phase currents are

$$\begin{bmatrix} I_3^a \\ I_3^b \\ I_3^c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} j0.6579 \\ -j2.6017 \\ j1.9438 \end{bmatrix} = \begin{bmatrix} 0 \\ 4.058\angle 165.93^\circ \\ 4.058\angle 14.07^\circ \end{bmatrix}$$

The fault current is

$$I_3(F) = I_3^b + I_3^c = 1.9732\angle 90^\circ$$



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**Q.3** **[20 marks]** **Achieved ILOS a1, a4, a5, b1, b2, b4, b5, b7, b11, c1, d1, d8**

a) For the network shown in Fig. 3, the reactance of each element is j0.2 pu. Select node 1 as the reference and form the incidence matrices: $\hat{A}$, $A$, $K$, $B$, $\hat{B}$, $C$, and $\hat{C}$ *(14 marks)*

b) Form the network matrices $Y_{BUS}$, $Y_{BR}$ and $Z_{LOOP}$. *(6 marks)*

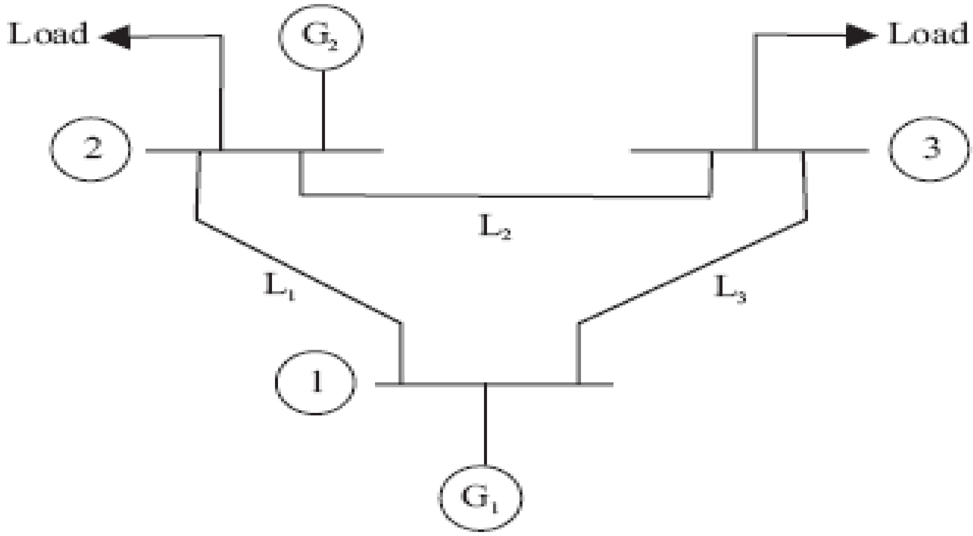
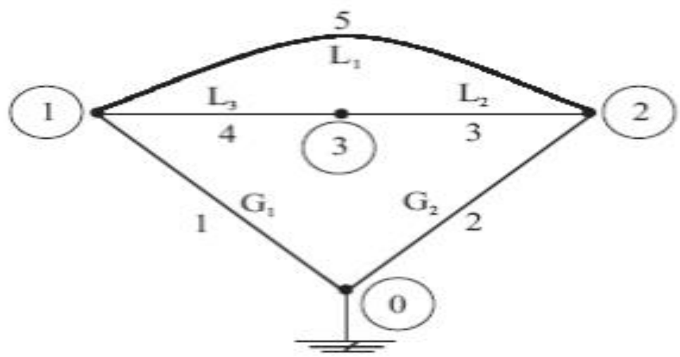



Fig. 3: Power system network

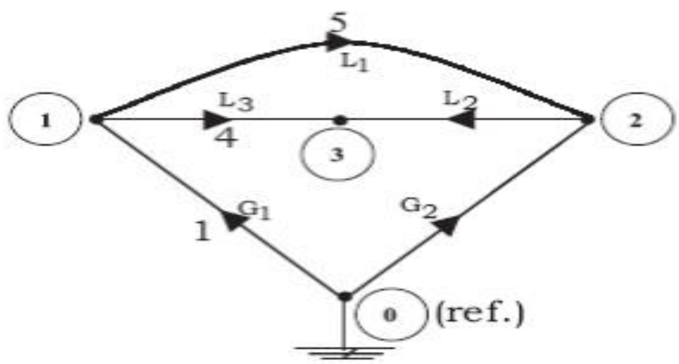
No. of elements, e = 5

No. of nodes, n = 4

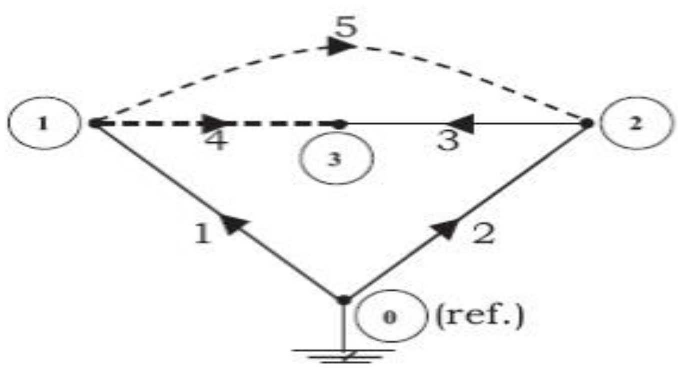
No. of buses = n – 1 = 4 – 1 = 3




Connected graph of power system.




Oriented graph of power system.




Tree of power system with tree branches, $T$ [1, 2, 3].

No. of branches, $b = n - 1 = 4 - 1 = 3$

No. of links, $\ell = e - b = e - (n - 1) = e - n + 1 = 5 - 3 = 2$

***Element node incidence matrix,*** $(\hat{A})$

$[\hat{A}] =$

| e \ n | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 1 | 1 | −1 | 0 | 0 |
| 2 | 1 | 0 | −1 | 0 |
| 3 | 0 | 0 | 1 | −1 |
| 4 | 0 | 1 | 0 | −1 |
| 5 | 0 | 1 | −1 | 0 |

***Bus incidence matrix, (A)***

$[A] =$

| e \ (n − 1) | 1 | 2 | 3 | |
|---|---|---|---|---|
| 1 | −1 | 0 | 0 | Branches ($A_b$) |
| 2 | 0 | −1 | 0 | |
| 3 | 0 | 1 | −1 | |
| 4 | 1 | 0 | −1 | Links ($A_\ell$) |
| 5 | 1 | −1 | 0 | |

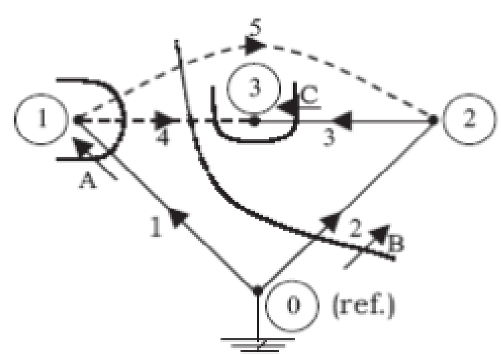
$$= \begin{bmatrix} A_b \\ \cdots \\ A_\ell \end{bmatrix}$$

***Bus cut set incidence matrix, (B)***

$[B] =$

| e \ Basic cut sets | A | B | C | |
|---|---|---|---|---|
| 1 | 1 | 0 | 0 | Branches ($B_b$ or $U_b$) |
| 2 | 0 | 1 | 0 | |
| 3 | 0 | 0 | 1 | |
| 4 | −1 | 1 | 1 | Links ($B_\ell$) |
| 5 | −1 | 1 | 0 | |

$$= \begin{bmatrix} U_b \\ \cdots \\ B_\ell \end{bmatrix}$$




Tree of power system with tree branches $T$[1, 2, 3].



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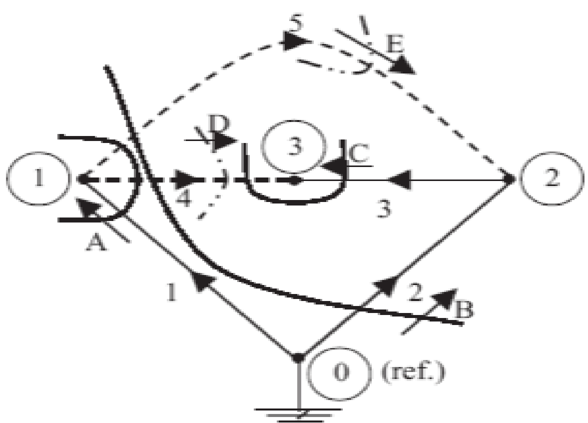
***Augmented cut set incidence matrix,*** $(\hat{B})$

$[\hat{B}] =$

| e \ e | A | B | C | D | E | |
|---|---|---|---|---|---|---|
| | Basic cut sets | | | Tie cut sets | | |
| 1 | 1 | 0 | 0 | 0 | 0 | Branches |
| 2 | 0 | 1 | 0 | 0 | 0 | |
| 3 | 0 | 1 | 1 | 0 | 0 | |
| 4 | −1 | 1 | 1 | 1 | 0 | Links |
| 5 | −1 | 1 | 0 | 0 | 1 | |

$$= \begin{bmatrix} U_b & O \\ \cdots & \cdots \\ B_\ell & U_\ell \end{bmatrix}$$



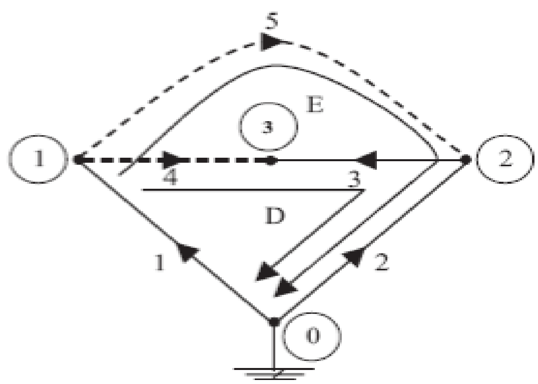

Tree of power system with tree branches, *T* [1, 2, 3].

***Basic loop incidence matrix, (C)***

$[C] =$

| e | D | E | |
|---|---|---|---|
| | Basic loops | | |
| 1 | 1 | 1 | Branches |
| 2 | −1 | −1 | |
| 3 | −1 | 0 | |
| 4 | 1 | 0 | Links |
| 5 | 0 | 1 | |

$$= \begin{bmatrix} C_b \\ \cdots \\ U_\ell \end{bmatrix}$$




Tree of power system with tree branches, *T* [1, 2, 3].



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**Q.4** **[10 marks] Achieved ILOS a1, a4, b1, b2, b5, c1, d1**

Write the performance equation of a three-phase element in impedance and admittance form for the following cases: (i) Balanced excitation (Stationary and Rotating element). (ii) Unbalanced excitation (Stationary and Rotating element).

### *Balanced excitation*

The excitation of any three-phase element is balanced when the source voltages or source currents of all phases are equal in magnitude and displaced from each other by 120°. For balanced excitation,

$$e_{pq}^{a,b,c} = \begin{bmatrix} e_{pq}^{a} \\ e_{pq}^{b} \\ e_{pq}^{c} \end{bmatrix} = \begin{bmatrix} 1 \\ a^2 \\ a \end{bmatrix} e_{pq}^{a} \quad \text{and} \quad j_{pq}^{a,b,c} = \begin{bmatrix} j_{pq}^{a} \\ j_{pq}^{b} \\ j_{pq}^{c} \end{bmatrix} = \begin{bmatrix} 1 \\ a^2 \\ a \end{bmatrix} j_{pq}^{a}$$

where

$$a = e^{j(2\pi/3)} = -\tfrac{1}{2} + j\tfrac{1}{2}\sqrt{3}$$

It follows that $a^3 = 1$, $a^2 + a + 1 = 0$, and $a^* = a^2$. The phase voltages and phase currents are balanced if the excitation of a balanced three-phase element is balanced. Then, the performance equation, in impedance form, for a stationary element is

$$\begin{bmatrix} 1 \\ a^2 \\ a \end{bmatrix} v_{pq}^{a} + \begin{bmatrix} 1 \\ a^2 \\ a \end{bmatrix} e_{pq}^{a} = \begin{bmatrix} z_{pq}^{s} & z_{pq}^{m} & z_{pq}^{m} \\ z_{pq}^{m} & z_{pq}^{s} & z_{pq}^{m} \\ z_{pq}^{m} & z_{pq}^{m} & z_{pq}^{s} \end{bmatrix} \begin{bmatrix} 1 \\ a^2 \\ a \end{bmatrix} i_{pq}^{a} \qquad (5.3.1)$$

and for a rotating element is

$$\begin{bmatrix} 1 \\ a^2 \\ a \end{bmatrix} v_{pq}^{a} + \begin{bmatrix} 1 \\ a^2 \\ a \end{bmatrix} e_{pq}^{a} = \begin{bmatrix} z_{pq}^{s} & z_{pq}^{m1} & z_{pq}^{m2} \\ z_{pq}^{m2} & z_{pq}^{s} & z_{pq}^{m1} \\ z_{pq}^{m1} & z_{pq}^{m2} & z_{pq}^{s} \end{bmatrix} \begin{bmatrix} 1 \\ a^2 \\ a \end{bmatrix} i_{pq}^{a} \qquad (5.3.2)$$

Both sides of equation (5.3.1) can be premultiplied by the conjugate transpose of

$$\begin{bmatrix} 1 \\ a^2 \\ a \end{bmatrix}$$

that is,

| | | |
|---|---|---|
| 1 | $a$ | $a^2$ |

to obtain

$$3v_{pq}^a + 3e_{pq}^a = 3(z_{pq}^s - z_{pq}^m)i_{pq}^a \tag{5.3.3}$$

Dividing by 3, equation (5.3.3) becomes

$$v_{pq}^a + e_{pq}^a = (z_{pq}^s - z_{pq}^m)i_{pq}^a$$

where $(z_{pq}^s - z_{pq}^m)$ is the positive sequence impedance, which is designated by $z_{pq}^{(1)}$. Thus, a balanced three-phase element with balanced excitation can be treated as a single-phase element in network problems. The power in the element is equal to three times the power per phase.

In a similar manner, equation (5.3.2) can be reduced to

$$v_{pq}^a + e_{pq}^a = (z_{pq}^s + a^2 z_{pq}^{m1} + a z_{pq}^{m2})i_{pq}^a$$

where $z_{pq}^s + a^2 z_{pq}^{m1} + a z_{pq}^{m2}$ is the positive sequence impedance.

The performance equation, in admittance form, for a stationary element is

$$i_{pq}^a + j_{pq}^a = (y_{pq}^s - y_{pq}^m)v_{pq}^a$$

and for a rotating element is

$$i_{pq}^a + j_{pq}^a = (y_{pq}^s + a^2 y_{pq}^{m1} + a y_{pq}^{m2})v_{pq}^a$$

***Unbalanced excitation***

When the excitation is unbalanced, the performance equation of a three-phase element can be reduced to three independent equations by diagonalizing the impedance matrix $z_{pq}^{a,b,c}$. Using a complex transformation matrix $T$ then the phase variables are expressed in terms of a new set of variables as follows:

$$\begin{aligned} v_{pq}^{a,b,c} &= T v_{pq}^{i,j,k} \\ e_{pq}^{a,b,c} &= T e_{pq}^{i,j,k} \\ i_{pq}^{a,b,c} &= T i_{pq}^{i,j,k} \end{aligned} \tag{5.3.4}$$

The complex power in the element is

$$S_{pq} = P_{pq} + jQ_{pq} = \{(i_{pq}^{a,b,c})^*\}^t e_{pq}^{a,b,c}$$

Substituting from equations (5.3.4),

$$S_{pq} = \{(i_{pq}^{i,j,k})^*\}^t (T^*)^t T e_{pq}^{i,j,k} \tag{5.3.5}$$



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The complex power in terms of the $i, j, k$ sequence variables is

$$S'_{pq} = [(i_{pq}^{i,j,k})^*]^t e_{pq}^{i,j,k} \tag{5.3.6}$$

If the complex powers $S_{pq}$ and $S'_{pq}$ are equal, that is, the selected transformation $T$ is power-invariant, then from equations (5.3.5) and (5.3.6),

$$(T^*)^t T = U = T(T^*)^t$$

Thus $T$ is a unitary matrix.

Substituting from equations (5.3.4) the performance equation (5.2.2) becomes

$$T(v_{pq}^{i,j,k} + e_{pq}^{i,j,k}) = z_{pq}^{a,b,c} T i_{pq}^{i,j,k} \tag{5.3.7}$$

Both sides of equation (5.3.7) can be premultiplied by $(T^*)^t$ to obtain

$$v_{pq}^{i,j,k} + e_{pq}^{i,j,k} = (T^*)^t z_{pq}^{a,b,c} T i_{pq}^{i,j,k}$$

It follows that

$$z_{pq}^{i,j,k} = (T^*)^t z_{pq}^{a,b,c} T \tag{5.3.8}$$

A similar transformation can be obtained for the performance equation in its admittance form.