Benha University Faculty of Engineering – Shoubra Preparatory Year for Master of Science in Engineering Physics		Final Term Exam Plasma Physics Date: 1 / 1 / 2019 Duration : 3 hours

- Answer all the following question
- Illustrate your answers with sketches when necessary
- No. of questions: 4 in two pages
- Total Mark: 60 Marks

Physical constants: Charge of an electron, $e = 1.6 \times 10^{-19}$ C
Mass of an electron, $m_e = 9.1 \times 10^{-31}$ kg
Mass of a proton, $m_p = 1.67 \times 10^{-27}$ kg
Boltzmann's constant, $K = 1.38 \times 10^{-23}$ J K^{-1}
Permittivity of free space, $\varepsilon_o = 8.85 \times 10^{-12}$ C^2 N^{-1} m^{-2}

Question (1) (20 marks)

(A) Give short notes on: **(9 marks)**

1) The plasma approximation.

Charged particles must be close enough together that each particle influences many nearby charged particles, rather than just interacting with the closest particle (these collective effects are a distinguishing feature of a plasma). The plasma approximation is valid when the number of charge carriers within the sphere of influence (called the *Debye sphere* whose radius is the Debye screening length) of a particular particle are higher than unity to provide collective behavior of the charged particles.

2) Bulk interaction of plasma.

The Debye screening length (defined above) is short compared to the physical size of the plasma. This criterion means that interactions in the bulk of the plasma are more important than those at its edges, where boundary effects may take place. When this criterion is satisfied, the plasma is quasineutral.

3) Plasma frequency.

The electron plasma frequency (measuring plasma oscillations of the electrons) is large compared to the electron-neutral collision frequency (measuring frequency of collisions between electrons and neutral particles). When this condition is valid, electrostatic interactions dominate over the processes of ordinary gas kinetics.

(B) Explain the meaning of "Degree of ionization of plasma". If a specific gas has number density of atoms as 5 x 10^{12} per cm^3 and when the gas subjected to high energy, the number density of ions was found to be 8 x 10^{11} per cm^3. What is the degree of ionization of the resultant plasma? **(6 marks)**

Degree of ionization of plasma.

The degree of ionization of a plasma is the proportion of atoms which have lost (or gained) electrons, and is controlled mostly by the temperature. Even a partially ionized gas in which as little as 1% of the particles are ionized can have the characteristics of a plasma (i.e., response to magnetic fields and high electrical conductivity). The degree of ionization, α is defined as $\alpha = n_i/(n_i + n_a)$ where n_i is the number density of ions and n_a is the number density of neutral atoms.

$n_i = 8 \times 10^{11}$ cm^{-3}

$n_i + n_a = 5 \times 10^{12}$ cm^{-3}

then, the degree of ionization can be given as:

$\alpha = n_i / (n_i + n_a) = 8 \times 10^{11} / 5 \times 10^{12} = 0.16 = 16\ \%$

(C) Discuss the concept of "Magnetization" and state the condition required for a plasma to be magnetized? **(5 marks)**

Plasma in which the magnetic field is strong enough to influence the motion of the charged particles is said to be magnetized. A common quantitative criterion is that a particle on average completes at least one gyration around the magnetic field before making a collision, i.e., $\omega_{ce}/\nu_{coll} > 1$, where ω_{ce} is the "electron gyrofrequency" and ν_{coll} is the "electron collision rate". It is often the case that the electrons are magnetized while the ions are not. Magnetized plasmas are *anisotropic*, meaning that their properties in the direction parallel to the magnetic field are different from those perpendicular to it. While electric fields in plasmas are usually small due to the high conductivity, the electric field associated with a plasma moving in a magnetic field is given by $\mathbf{E} = -\mathbf{v} \times \mathbf{B}$ (where **E** is the electric field, **v** is the velocity, and **B** is the magnetic field), and is not affected by Debye shielding.

Question (2) (20 marks)

(A) Plasma has characteristic length, use the equations of electrostatic field to deduce the potential produced by specific plasma at distance r from its center? Then find the mathematical formula of the Debye length? **(7 marks)**

First, consider a positive charge q all by itself. The potential at a distance r from the charge is

$$\phi = \frac{q}{4\pi\varepsilon_0 r}.$$

Now, consider a positive charge q in the middle of a plasma. It attracts electrons into its vicinity and repels positive ions. We will calculate ϕ for this case.

If we allow the particle to have both kinetic and potential energy, the probability factor becomes $\exp\left(-\frac{\frac{1}{2}mv^2 + q\phi}{kT}\right)dv_x dv_y dv_z$. ϕ depends on position so the probability depends on position.

The particle density is given by $n = \int f(v)dv_x dv_y dv_z$ so $n \propto \exp\left(-\frac{q\phi}{kT}\right)$

for electrons $n_e = n_0 \exp\left(-\frac{-e\phi}{kT}\right)$

for ions (we will suppose they are singly-ionized) $n_i = n_0 \exp\left(-\frac{e\phi}{kT}\right)$

Gauss' Law can be written as

$$\nabla.\mathbf{E} = \frac{\sigma}{\varepsilon_0}$$

$\mathbf{E} = -\nabla\phi$ so

$$-\nabla^2\phi = \frac{\sigma}{\varepsilon_0}.$$

This is *Poisson's equation*.

The charge density is $\sigma = -en_e + en_i = en_0\left(-\exp\frac{e\phi}{kT} + \exp\frac{-e\phi}{kT}\right)$.

Assume that this potential term is very small, $e\phi << kT$

$$\sigma \cong -en_0\left(1 + \frac{e\phi}{kT}\right) + en_0\left(1 - \frac{e\phi}{kT}\right) = -\frac{2n_0 e^2\phi}{kT}.$$

I am going to use spherical coordinates (and assume spherical symmetry)

$$\nabla^2\phi \equiv \frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\phi}{dr}\right).$$

Poisson's equation becomes

$$-\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\phi}{dr}\right) = -\frac{2n_0e^2\phi}{\varepsilon_0 kT}$$

with solution

$$\phi = \frac{q}{4\pi\varepsilon_0 r}\exp\left(-\frac{r}{\sqrt{\frac{\varepsilon_0 kT}{2n_0e^2}}}\right).$$

The potential falls away exponentially.

Call $\boxed{\lambda_D = \sqrt{\frac{\varepsilon_0 kT}{n_0 e^2}}}$ the *Debye length* then

$$\phi = \frac{q}{4\pi\varepsilon_0 r}\exp\left(-\frac{\sqrt{2}r}{\lambda_D}\right).$$

Beyond a few Debye lengths, shielding by the plasma is quite effective and the potential due to our charge is negligible.

(B) For the radio-frequency discharge in the Senior Physics Lab, T = 3 eV, n_e = 10^{17} m^{-3} and diameter about 100 mm, calculate:

1) The Debye length.
2) The plasma frequency.
3) The number of electrons in a Debye sphere. **(6 marks)**

K T = 3 eV = 3 x $1.6\text{x}10^{-19}$ J $n_o = n_e = 10^{17}$ m^{-3}

1)

$$\lambda_D = \sqrt{\frac{\varepsilon_0 kT}{n_0 e^2}}$$

= $4.1\text{x}10^{-5}$ m

2)

$$f_{pe} = 8.98\sqrt{n_e}$$

= $2.8\text{x}10^{9}$ Hz

3) N_D = 4/3 π $(4.1\text{x}10^{-5})^3$ x 10^{17}

= $2.8\text{x}10^{4}$

(C) Differentiate clearly between the two concepts of “hot plasma” and “cold plasma”? If you told that a plasma of 1 million K is contained in a steel vessel without melting it, how can you explain that? **(7 marks)**

temperature is an important factor in plasma formation as the kinetic energy of a plasma particle is considerably higher than its potential, where charged particles travel at high speeds. If the potential were greater than the kinetic, then the plasma state would be destroyed as the ions and electrons would want to clump together into bound states—atoms. This is why plasmas typically arise at very high temperatures. In most cases the electrons are close enough to thermal equilibrium that their temperature is relatively well-defined, even when there is a significant deviation from a Maxwellian energy distribution function, for example, due to UV radiation, energetic particles, or strong electric fields.

Based on the relative temperatures of the electrons, ions and neutrals, plasmas are classified as "thermal" or "non-thermal". Thermal plasmas have electrons and the heavy particles at the same temperature, i.e., they are in thermal equilibrium with each other. Non-thermal plasmas on the other hand have the ions and neutrals at a much lower temperature, (normally room temperature), whereas electrons are much "hotter". Temperature controls the degree of plasma ionization. In particular, plasma ionization is determined by the "electron temperature" relative to the ionization energy, (and more weakly by the density), in a relationship called the Saha equation. A plasma is sometimes referred to as being "hot" if it is nearly fully ionized, or "cold" if only a small fraction, (for example 1%), of the gas molecules are ionized, but other definitions of the terms "hot plasma" and "cold plasma" are common. Even in a "cold" plasma, the electron temperature is still typically several thousand degrees Celsius. Plasmas utilized in "plasma technology" ("technological plasmas") are usually cold in this sense.

Question (3) (20 marks)

(A) Compare the main characteristics which differentiate the plasma state and ordinary gaseous phase of matter? **(6 marks)**

Property	**Gas**	**Plasma**
Electrical Conductivity	**Very low** **Air is an excellent insulator until it breaks down into plasma at electric field strengths above 30 kilovolts per centimeter.**	**Usually very high** **For many purposes, the conductivity of a plasma may be treated as infinite.**
Independently acting species	**One** **All gas particles behave in a similar way, influenced by gravity and by collisions with one another.**	**Two or three** **Electrons, ions, protons and neutrons can be distinguished by the sign and value of their charge so that they behave independently in many circumstances, with different bulk velocities and temperatures, allowing phenomena such as new types of waves and instabilities.**
Velocity distribution	**Maxwellian** **Collisions usually lead to a Maxwellian velocity distribution of all gas particles, with very few relatively fast particles.**	**Often non-Maxwellian** **Collisional interactions are often weak in hot plasmas and external forcing can drive the plasma far from local equilibrium and lead to a significant population of unusually fast particles.**
Interactions	**Binary** **Two-particle collisions are the rule, three-body collisions extremely rare.**	**Collective** **Waves, or organized motion of plasma, are very important because the particles can interact at long ranges through the electric and magnetic forces.**

(B) Discuss the process of ionization produced by electric fields and explain with necessary graphs the principal stages of gas discharge? **(6 marks)**

Just like the many uses of plasma, there are several means for its generation, however, one principle is common to all of them: there must be energy input to produce and sustain it. For this case, plasma is generated when an electrical current is applied across a dielectric gas or fluid (an electrically non-conducting material) as can be seen in the image below, which shows a discharge tube to be considered as a simple example (DC used for simplicity).Cascade process of ionization. Electrons are 'e−', neutral atoms 'o', and cations '+'.

The potential difference and subsequent electric field causes the bound electrons (negative) to be pulled toward the anode (positive electrode) while the nucleus (positive) is pulled to the cathode (negative electrode). As the voltage is increased, the current stresses the material (by electric polarization) beyond its dielectric limit (termed strength) into a stage of electrical breakdown, marked by a spark, where the material transforms from being an insulator into a conductor (as it becomes increasingly ionized). This is a stage of avalanching ionization, where collisions between electrons and neutral gas atoms, create more ions and electrons (as can be seen in the figure on the right). The first impact of an electron on an atom results in one ion and two electrons. Therefore, the number of charged particles increases rapidly (in the millions) only "after about 20 successive sets of collisions", mainly due to a small mean free path (average distance travelled between collisions).

With ample current density and ionization, this forms a luminous electric arc (essentially lightning) between the electrodes. Electrical resistance along the continuous electric arc creates heat, which ionizes more gas molecules (where degree of ionization is determined by temperature), and as per the sequence: solid-liquid-gas-plasma, the gas is gradually turned into a thermal plasma. A thermal plasma is in thermal equilibrium, which is to say that the temperature is relatively homogeneous throughout the heavy particles (i.e. atoms, molecules and ions) and electrons. This is so because when thermal plasmas are generated, electrical energy is given to electrons, which, due to their great mobility and large numbers, are able to disperse it rapidly and by elastic collision (without energy loss) to the heavy particles.

(C) Write down on <u>only</u> one of the following subjects: **(8 marks)**

1) Laser-induced breakdown spectroscopy and its applications.
2) The usage of plasma in recycling and waste disposal processes.

<u>Answer:</u> the Answer is accepted through your external readings

GOOD LUCK

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