Benha University
Faculty of Engineering- Shoubra
Credit hour Program
Final Examination

Subject: Mathematics 6
Code: EEC 325
Date: 1st Jan. 2019
Duration : 2 hours

- **Answer all the following questions**
- **Illustrate your answers with sketches when necessary.**
- **The Exam. Consists of one Page**

- **No. of Questions: 4**
- **Total Mark:** 40 Marks

1) In a factory we have four machines producing 1000, 1200, 1800, 2000 items per day with defects 1%, 5%, 5%, 1% respectively, find:

i) The probability of selecting a defective item.

ii) The probability that this defective item is produced by third machine.

2) If a r.v. X takes the values 1, 2, 3, 4 such that 2 P(X = 1) = 5 P(X = 2) = P(X = 3) = 3P(X = 4). Find the probability distribution of X, also find mean and variance.

3) $f(x) = \begin{cases} c(1 - x^2) & -1 < x < 1 \\ 0 & \text{otherwise} \end{cases}$, find c, median, $E|X|$ and $p(-1/2 < x < 1/3)$.

4) Find the constants of the curve $y(x) = \dfrac{1}{a + bx}$ that fit the following data: (-1,2), (3,4), (6,9).

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Model answer

Answer of Question 1

i) Let the four machines are A, B, C, D respectively such that P(A) = 1000/6000 = 1/6, P(B) = 1200/6000 = 1/5, P(C) = 1800/6000 = 3/10, P(D) = 2000/6000 = 1/3, P(d/A)=0.01, P(d/B)=0.05, P(d/C) = 0.05, P(d/D) = 0.01

P(d) = P(d/A) P(A)+ P(d/B) P(B)+ P(d/C) P(C)+ P(d/D) P(D)=0.03

ii) The probability that this defective item is produced by third machine P(C/d) = $\frac{P(d/C)P(C)}{P(d)} = \frac{(0.05)(0.3)}{0.03} = 0.5$.

Answer of Question 2

Let 2P(X = 1)= 3P(X = 2) = P(X = 3) = 5P(X = 4) = 30 P, therefore P(X = 1) = 15P, P(X=2) = 10P, P(X = 3) = 30P, P(X = 4) = 6P, but P(X=1) + P(X=2) + P(X=3) + P(X=4) = 1, thus P = 1/61

X	1	2	3	4
fX)	15/61	10/61	30/61	6/61

E(X) = (1/61)[15 + 20 + 90 + 24] = 149/61

$E(X^2)$ = (1/61)[15 + 40 + 270 + 96] = 421/61

Var(X) = $(421/61) - (149/61)^2$

Answer of Question 3

$$c\int_{-1}^{1}(1-x^2)\,dx = 1 \Rightarrow 2c[x-\frac{x^3}{3}]\Big|_0^1 = 1 \Rightarrow c = 3/4$$

$$E|X| = \frac{3}{4}[\int_{-1}^{0} -x(1-x^2)dx + \int_{0}^{1} x(1-x^2)dx]$$

$$p(-1/2 < x < 1/3) = \frac{3}{4}\int_{-1/2}^{1/3}(1-x^2)\,dx = \frac{3}{4}[x-\frac{x^3}{3}]\Big|_{-1/2}^{1/3} = \frac{3}{4}[\frac{1}{3}-\frac{1}{81}+\frac{1}{2}-\frac{1}{24}]$$

To get the median μ_x

$$\frac{3}{4}\int_{-1}^{\mu_x}(1-x^2)\,dx = 0.5 \Rightarrow \frac{3}{4}[x-\frac{x^3}{3}]\Big|_{-1}^{\mu_x} = \frac{3}{4}[\mu_x - \frac{{\mu_x}^3}{3} + 1 - \frac{1}{3}] = 0.5$$

Answer of Question 4

If we consider the function $1/y = a + bx$ such that

$\sum_{i=1}^{N} 1/y_i = aN + b\sum_{i=1}^{N} x_i$, $\sum_{i=1}^{N} x_i/y_i = a\sum_{i=1}^{N} x_i + b\sum_{i=1}^{N} x_i^2$, where $N = 3$,

$\sum_{i=1}^{3} 1/y_i = 0.86$, $\sum_{i=1}^{3} x_i = 8$, $\sum_{i=1}^{3} x_i^2 = 46$, $\sum_{i=1}^{3} x_i/y_i = 0.92$, therefore:

0.86= 3a + 8b, 0.92 = 8a + 46b, from which we get a and b.