Benha University
Faculty of Engineering- Shoubra
Credit hour Programs
Final Examination

Subject: Mathematics 2
Code: EMP 102
Date: 5th Jan. 2019
Duration : 2 hours

- **Answer all the following questions**
- **Illustrate your answers with sketches when necessary.**
- **The Exam. Consists of one Page**
- **No. of Questions: 2**
- **Total Mark:** 40 Marks

Question 1 [21]

1- Find the equation of the circle which intersects the circles $x^2 + y^2 + 2x - 2y + 1 = 0$ & $x^2 + y^2 + 4x - 4y + 3 = 0$ orthogonally, whose center lies on the line $3x - y - 2 = 0$ [7]

2- Find coordinates of vertex, focus, directrix for $x^2 - 2x - 4y - 15 = 0$ [7]

3- Find k such that $2x^2 - 4xy + 2y^2 + x + k\,y - 1 = 0$ represent pair of straight lines, and find the distance between the two lines. [7]

Question 2 [19]

1- Compute the arc length of the graph of $f(x) = x^{3/2}$ over [0, 1] [5]

2- Solve the differential equations $y'' = x + \cos x$, $y(0) = 2$, $y'(0) = 4$ [5]

6- Evaluate the integrals [9]

a) $\int \frac{(x+4)dx}{\sqrt{x^2+6x+10}}$, b) $\int x^3 \cos(4x)\,dx$, c) $\int 3(8y-1)\,e^{(4y^2-y)}\,dy$

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Answer of Q1

1- Let equation of the required circle be $x^2 + y^2 + 2g x + 2f y + c = 0$.. (i)

Since this circle cuts the circles $x^2 + y^2 + 2x - 2y + 1 = 0$ and $x^2 + y^2 + 4x - 4y + 3 = 0$ orthogonally, we get $2(g. 1 + f. (-1)) = c + 1 \Rightarrow 2g - 2f - c - 1 = 0$ ...(ii)

And $2(g. (2) + f. (-2)) = c + 3 \Rightarrow 4g - 4f - c - 3 = 0$ ... (iii)

As center of the required circle $(-g, -f)$ lies on $3x - y - 2 = 0$, we get $-3g + f - 2 = 0 \Rightarrow 3g - f + 2 = 0$ ... (iv)

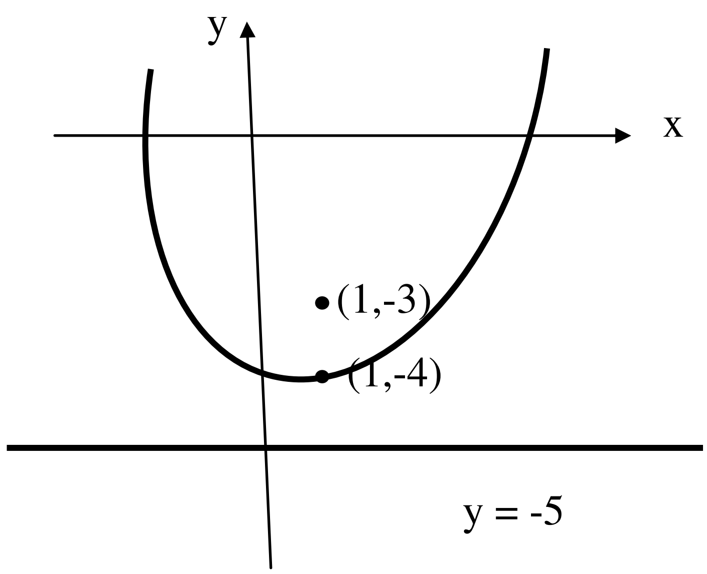
Subtracting (ii) from (iii), so $2g - 2f - 2 = 0 \Rightarrow g - f - 1 = 0$ ...(v)

Solving (iv) and (v) simultaneously, then $g = -\frac{3}{2}$ and $f = -\frac{5}{2}$.

From (ii), we get $c = 2g - 2f - 1 = -3 + 5 - 1 = 1$.

Substituting these values of g, f and c in (i), we get $x^2 + y^2 - 3x - 5y + 1 = 0$, which is the equation of the required circle.

2- By completing square, we will get
$(x-1)^2 - 1 - 4y - 15 = 0 \Rightarrow (x-1)^2 = 4y + 16 = 4(y + 4)$
$4p = 4 \Rightarrow p = 1$, h = 1, k = -4, therefore vertex v = (1,-4), focus f = (1,-3) and directrix y = -5.



3) $a = 2, h = -2, b = 2, g = ½, f = k/2, c = -1$ and since $h^2 = ab = 4$, therefore the two lines are parallel. The above equation represents pair of straight lines if

$$\begin{vmatrix} 2 & -2 & 1/2 \\ -2 & 2 & k/2 \\ 1/2 & k/2 & -1 \end{vmatrix} = 0 \Rightarrow 2[-2 - (k^2/4)] + 2[2 - (k/4)] - [k + 1] / 2 = 0 \Rightarrow [k + 1]^2 = 0$$

$\Rightarrow k = -1$, therefore the equation of the pair of st. lines is $2x^2 - 4xy + 2y^2 + x - y - 1 = 0 \Rightarrow (x - y + c_1)(2x - 2y + c_2) = 0$, therefore $2x^2 - 4xy + 2y^2 + (c_2 + 2c_1)x - (2c_1 + c_2) y + c_1c_2 = 0$ and by comparing coefficients of x and the constant such that:

$$c_2 + 2c_1 = 1, \qquad c_1 c_2 = -1$$

By solving the two equations simultaneously, we get $c_1 = 1$ and $c_2 = -1$ and therefore the two lines are $x - y + 1 = 0$ and $2x - 2y - 1 = 0$.

If the unknown is the coefficient of x or y, then we can get the unknown coefficient by separating the two lines and by comparing the coefficients, we can get the unknown coefficient. For the above example, we can get the two parallel lines separately as follows:

$2x^2 - 4xy + 2y^2 + (c_2 + 2c_1)\, x - (2c_1 + c_2)\, y + c_1 c_2 = 0$, therefore and by comparing coefficients of x and the constant such that $c_2 + 2c_1 = 1, \quad c_1 c_2 = -1$.

By solving the two equations simultaneously, we get $c_1 = 1$ & $c_2 = -1$, therefore the two lines are $L_1 : x - y + 1 = 0$ and $L_2: 2x - 2y - 1 = 0$, but $k = -(2c_1 + c_2) = -1$.

To get the shortest distance between the two lines, put $x = 0$ in L_1, therefore $A(0, 1)$ satisfy L_1 and drop a perpendicular line from $(0,1)$ on L_2 at point B, so that $AB = \dfrac{|-3|}{\sqrt{2^2 + 2^2}} = \dfrac{3}{2\sqrt{2}}$.

Answer of Q2

1- $f'(x) = (3/2)\, x^{1/2}$, therefore $\mathbf{L = \int_0^1 \sqrt{1 + [f'(x)]^2}\,dx = \int_0^1 \sqrt{1 + \frac{9}{4}x}\,dx}$

$$\mathbf{= \frac{8}{27}\left(1 + \frac{9}{4}x\right)^{3/2}\Big|_0^1 = \left(\frac{13}{4}\right)^{3/2} - 1}$$

2- y\`(x)= $x^2/2$ + sin x +c , but at y\`=4, x = 0, therefore c = 4, therefore

y\`(x) = $x^2/2$ + sin x +4, integrate, we will get y(x)= $x^3/6$ – cos x + 4x+d , but at y=2 , x=0, therefore d= 3, thus the solution is y(x)= $x^3/6$ – cosx + 4x+3

3-a) By completing square, we get $\int \frac{(x+4)dx}{\sqrt{x^2+6x+10}} = \int \frac{(x+3+1)dx}{\sqrt{(x+3)^2+1}}$

$$= \frac{1}{2}\int \frac{2(x+3)dx}{\sqrt{(x+3)^2+1}} + \int \frac{dx}{\sqrt{(x+3)^2+1}} = \sqrt{(x+3)^2+1} + \sinh^{-1}(x+3)$$

b) $\int x^3 \cos(4x)\,dx = x^3 (\frac{\sin(4x)}{4}) - 3x^2(\frac{-\cos(4x)}{16}) + 6x(\frac{-\sin(4x)}{64}) - 6(\frac{\cos(4x)}{256})$

c) $\int 3(8y-1)\, e^{(4y^2-y)}\, dy = 3e^{(4y^2-y)}$