Benha University
Faculty of Engineering (Shoubra)
Engineering Mathematics and Physics Department

Probability and random processes
Code: ENG 506 Jan. 2019
Time allowed: 3 hours
Scores : 60 marks

Question 1 [30]

a- A card is drawn at random from a standard deck of playing cards. What is the Prob. that the card is less than a 7 given: A) The card is not a 2, B) The card is a heart, C) The card is a 3 or 4 **[9]**

b- Suppose that a number of inquiries arriving at a certain interactive system with arrival rate 12 inquiries per minute. Find the probability of at least 2 inquiries arriving in 3 minute interval. **[7]**

c- When Justin is goal-keeper, Shaunie manages to score an average of once for every 10 shots he takes. If Shaunie takes 12 shots, find the following probability that he scores at most twice. **[7]**

d- If $f(x) = c\, x^2 e^{-2x}$ is P.d.f., $x > 0$, find c, mean and variance**.** **[7]**

Question 2 [20]

a) A fair coin is tossed 3 times, X is the N^o of heads that come up on the first 2 tosses and Y is the N^o of heads that come up on tosses 2, 3. Construct the joint distribution and find marginal of X and Y, also find the variance of X , Y & P [(X + Y) >1, X > Y]. **[10]**

b) $f(x) = \begin{cases} c(1-x^2) & -1 < x < 1 \\ 0 & \text{otherwise} \end{cases}$, find c, median, $E|X|$ and $p(-1/2 < x < 1/3)$. **[10]**

Question 3 [10]

a) If $f(x) = c\, e^{-2x}$ is P.d.f., find E(x) and standard deviation **[5]**

b) Given a Binomial distribution with variance = 3 and mean = 6, find the number of trials. **[5]**

Good luck **Board of examiners Dr. eng Khaled El Naggar**

Model answer

Answer of question 1

a - D = {card is less than a 7} = {24 cards}

A = {card is not a 2} = {48 cards}, therefore P(D/A) = 20/48 = 5/12

B = {card is a heart} = {13 cards}, therefore P(D/B) = 6/13,

C = {card is a 3 or 4} = {8 cards}, therefore P(D/C) = 1.

b- X: The number of inquiries arriving at a certain interactive such that $\lambda = 12(3) = 36$, thus

$P(X \geq 2) = 1 - P(X \leq 1) = 1 - \sum_{x=0}^{1} \frac{e^{-\lambda}\lambda^x}{x!} = 1 - \sum_{x=0}^{1} \frac{e^{-36}36^x}{x!} = 1 - e^{-36} - 36e^{-36}$

c- By using Binomial distribution, n = 12, p = 0.1, therefore

$p(x \leq 2) = \sum_{x=0}^{2} {}^{12}c_x (0.1)^x (0.9)^{12-x}$

d- Since $f(X) = 4x^2e^{-2x}$ is gamma distribution with $\alpha = 3$ & $\beta = 2$, thus $E(X) = \alpha / \beta = 3/2$ and variance is $\alpha / \beta^2 = 3/4$

Answer of question 2

a)

Y \ X	0	1	2	$f_1(x)$
0	1/8	1/8	0	2/8
1	1/8	2/8	1/8	4/8
2	0	1/8	1/8	2/8
$f_2(y)$	2/8	4/8	2/8	1

$E(X) = 0(2/8) + 1(4/8) + 2(2/8) = 1$, $E(Y) = 0(2/8) + 1(4/8) + 2(2/8) = 1$, $E(X^2) = 1(4/8) + 4(2/8) = 3/2$, $E(Y^2) = 1(4/8) + 4(2/8) = 3/2$, $Var(X) = Var(Y) = 1/2$.

$P[(X + Y) > 1, X > Y] = P(2,0) + P(2,1) = 1/8$

b) $c\int_{-1}^{1}(1-x^2)dx = 1 \Rightarrow 2c[x - \frac{x^3}{3}]\Big|_0^1 = 1 \Rightarrow c = 3/4$

$$E|X| = \frac{3}{4}[\int_{-1}^{0} -x(1-x^2)dx + \int_{0}^{1} x(1-x^2)dx]$$

$$p(-1/2 < x < 1/3) = \frac{3}{4}\int_{-1/2}^{1/3}(1-x^2)dx = \frac{3}{4}[x - \frac{x^3}{3}]\Big|_{-1/2}^{1/3} = \frac{3}{4}[\frac{1}{3} - \frac{1}{81} + \frac{1}{2} - \frac{1}{24}]$$

To get the median μ_x

$$\frac{3}{4}\int_{-1}^{\mu_x}(1-x^2)dx = 0.5 \Rightarrow \frac{3}{4}[x - \frac{x^3}{3}]\Big|_{-1}^{\mu_x} = \frac{3}{4}[\mu_x - \frac{\mu_x^3}{3} + 1 - \frac{1}{3}] = 0.5$$

Answer of Question 3

a) f(x) is exponential distribution such that $f(x,\lambda) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$, thus $c = 2 = \lambda$ and the mean of X is given by $E(X) = \frac{1}{\lambda} = 0.5$ and the variance of X is given by $Var(X) = \frac{1}{\lambda^2} = 0.25$

b) Since for binomial distribution the expected value = n p = 6 and the variance = n p q = 3 and by dividing variance / expected value = q = 0.5 , therefore p = 1 – q = 0.5.
Hence the N^o of trials = n = 12

1. **Overall aims of course**

   By the end of the course the students will be able to:

   1. T o teach the students the essential information about Multi dimensional random variable**.**
   2. To teach the students characteristic functions :Limit theorem for characteristic functions

3. To teach the students converse limit theorem – Stochastic process
4. To teach the students Functions of random variables
5. To teach the students distribution function technique , transformation technique and moment generating function technique
6. To teach the students Markow chains , reliability theory and measures of reliability.

2. Intended Learning outcomes of Course (ILOs)

Knowledge and Understanding:

2.1.5 List the principles and fundamentals of probability and statistics [**Q1, Q2**]

2.1.6 Define the basics and the ethics of of random variables and probability distributions. [**Q2, Q3**]

Intellectual Skills

2.2.1 Analyze and assess information of random variables [**Q2**]
2.2.2 Solve problems of probability distributions. [**Q2**]

Professional and Practical Skills

2.3.2 Write and evaluate professional reports in probability and statistics and its applications [**Q1**]
2.3.3 Assess methods and current tools for treating, solving and analyzing real problems. [**Q2**]

a. General and Transferable Skills

2.4.2 Use information technology to develop modeling, and analyzing real problems

2.4.8 Use self learning and continuous research for recent topics related to this subject