Benha University Faculty of Engineering- Shoubra (Mechanical)Engineering Department		Final term exam Date: 21-5-2014 Mathematics 2B Code: EMP133 Duration : 3 hours
Answer all the following questions	No. of questions : 3	Total Mark: 100 points

Question 1 **[30 marks]**

Evaluate the following integrals

1) $\int_0^{\infty} \frac{t^{c+1}}{(1+t^2)^2} dt$ 2) $\int_0^1 \frac{t^{3c-m}}{\sqrt[3]{(1-t^3)}} dt$ 3) $\int_0^{\infty} a^{-m x^n} dx$

4) $\int_0^{\infty} (\frac{e^{2t} - \cos 3t}{t})\, e^{-3t}\, dt$ 5) $\int_{-\pi/6}^{\pi/3} (\sqrt{3}\sin\theta + \cos\theta)^{1/4} d\theta$ 6) $\int_2^{\infty} e^{-x^2+4x-4} dx$

Question 2 **[40 marks]**

I) If three fair coins are tossed once and the random variable is the number of heads facing up. Discuss the law of distribution and find the mean, variance, mode and median.

II) The random variable X has probability density function $f(x) = ax+bx^3$, $0 < x < 1$ and equal zero otherwise, find a,b if E(X) = 0.3, then find standard deviation.

III) A standard deck of 52 cards mixed well, one card is drawn at random, if A is the event that an ace is taken out and B is the event that a red card is taken out. Are A and B independent events?

IV) In a factory we have four machines producing 1000, 1200, 1800, 2000 items per day with defects 1%, 5%, 5%, 1% respectively, find :

a) The probability of selecting a defective item.

b) The probability that this defective item is produced by third machine.

Question 3 **[30 marks]**

I) Solve the following system of differential equations using Laplace transform

$$x` + y` - y = 2t + e^t, \quad x + y` - y = t^2 + 1 + e^t, \quad x(0) = 1,\ y(0) = 0$$

II) Solve the D.E. y``- xy`+ y=0 using series solution about x = 2

III) Find a straight line that best fit the data (-1,3), (1,7), (3,2)

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Model answer

Answer of Q1

1)Put $y = t^2$, therefore $dt = \frac{1}{2} y^{-\frac{1}{2}} dy$, therefore $\int_0^\infty \frac{t^{c+1}}{(1+t^2)^2} dt = \frac{1}{2}\int_0^\infty \frac{y^{c/2}}{(1+y)^2} dy = \frac{1}{2}\beta(m,n)$,

where m-1 = c/2, m+n = 2

2) Put $y = t^3$, therefore $dt = \frac{1}{3} y^{-\frac{2}{3}} dy$, therefore $\int_0^1 \frac{t^{3c-m}}{\sqrt[3]{(1-t^3)}} dt = \frac{1}{3}\int_0^1 \frac{y^{(3c-m)/3} y^{-2/3}}{\sqrt[3]{(1-y)}} dy$

3) $\int_0^\infty a^{-mx^n} dx = \int_0^\infty e^{-mx^n \ln a} dx$, so that put $y = mx^n \ln a \Rightarrow dy = mnx^{n-1} \ln a \, dx \Rightarrow$

$dx = \frac{y^{\frac{1}{n}-1} dy}{n(m\ln a)^{1/n}}$, therefore $\int_0^\infty e^{-mx^n \ln a} dx = \int_0^\infty \frac{y^{\frac{1}{n}-1} dy}{n(m\ln a)^{1/n}} e^{-y} dy = \frac{\Gamma(\frac{1}{n})}{n(m\ln a)^{1/n}} = \frac{\Gamma(\frac{1}{n}+1)}{(m\ln a)^{1/n}}$.

4) $\int_0^\infty (\frac{e^{2t} - \cos 3t}{t}) e^{-3t} dt = L\{ \frac{e^{2t} - \cos 3t}{t} \}_{s=3} = \int_s^\infty (\frac{1}{s-2} - \frac{s}{s^2+9})ds_{\ s=3} = \ln[\frac{\sqrt{s^2+9}}{s-2}]_{s=3} = \frac{1}{2}\ln 18$

5) $\int_{-\pi/6}^{\pi/3} (\sqrt{3}\sin\theta + \cos\theta)^{1/4} d\theta == 2\int_{-\pi/6}^{\pi/3} [\cos(\pi/6)\sin\theta + \cos(\pi/6)\cos\theta]^{1/4} d\theta$

$= 2\int_{-\pi/6}^{\pi/3} (\frac{\sqrt{3}}{2}\sin\theta + \frac{1}{2}\cos\theta)^{1/4} d\theta$, put $y = \theta + \pi/6$, thus

$\int_{-\pi/6}^{\pi/3} (\sqrt{3}\sin\theta + \cos\theta)^{1/4} d\theta = \int_{-\pi/6}^{\pi/3} [\sin(\theta + \pi/6)]^{1/4} d\theta = \int_0^{\pi/2} [\sin y]^{1/4} dy = \frac{1}{2}\beta(m,n)$, thus

2m-1 = 1/4 , 2n-1 =0 $\Rightarrow$ m = 5/8, n = 1/2.

6) $\int_2^\infty e^{-x^2+4x-4} dx = \int_2^\infty e^{-(x-2)^2} dx$, Put $y = (x-2)^2 \Rightarrow dy = 2(x-2)\,dx \Rightarrow dx = -\frac{dy}{2\sqrt{y}}$

Therefore $\int_2^\infty e^{-(x-2)^2} dx = \frac{1}{2}\int_0^\infty y^{-1/2} e^{-y} dy = \frac{\sqrt{\pi}}{2}$.

Answer of Q2

x	0	1	2	3
f(x)	1/8	3/8	3/8	1/8

E(X) = 0(1/8) + 1(3/8) + 2(3/8) + 3(1/8) = 3/2, $E(X^2)$ = 0(1/8) + 1(3/8) + 4(3/8) + 9(1/8) = 3,

Var(X) = $E(X^2) – [E(X)]^2$ = 3 – 9/4 = ¾.

Mode = {1, 2}

<u>C.d.f.</u>

x	0	1	2	3
f(x)	1/8	4/8	7/8	1

Median = {1}

II) Since $\int_0^1 x(ax+bx^3)dx = 0.3$, therefore $[a\frac{x^3}{3} + b\frac{x^5}{5}]_0^1 = 0.3$, thus 5a + 3b = 4.5 , but $\int_0^1 (ax+bx^3)dx = 1$, therefore $[a\frac{x^2}{2} + b\frac{x^4}{4}]_0^1 = 1$, thus 2a + b =4, so a = 7.5 and b = -11

Since E(2X) = 0.6, therefore E(X) = 0.3 and $E(X^2)$ = $\int_0^1 x^2(7.5x\text{-}11x^3)dx =$ $[\frac{7.5x^4}{4} - \frac{11x^6}{6}]_0^1$ = 1/24, therefore V(X) = $E(X^2)$ - $[E(X)]^2$ = -0.0483.

III) P(A) = 4/52 = 1/13, P(B) = 26/52 = 1/2 and $P(A \cap B)$ = 2/52 = 1/26 = P(A)P(B), therefore A and B are independent events.

IV) Let the defective event is F and the probability of machines A,B,C,D are 1/6, 1/5, 3/10, 1/3 respectively, also P(F/A) = 0.01, P(F/B) = 0.05, P(F/C) = 0.05, P(F/D) = 0.01.

Therefore

i- P(F) = P(F/A)P(A) + P(F/B)P(B) + P(F/C)P(C) + P(F/D)P(D) = 0.01(1/6) + 0.05(1/5)

+ 0.05(3/10) + 0.01(1/3) = 0.03

ii- $P(C/F) = \frac{P(F/C)P(C)}{P(F)} = \frac{0.05(3/10)}{0.03} = 0.5$

Answer of Q3

After taking Laplace to both equations, we get

$SX(S) + (S-1)Y(S) = \frac{2}{S^2} + \frac{1}{S-1} + 1$ and $X(S) + Y(S)(S-1) = \frac{2}{S^3} + \frac{1}{S} + \frac{1}{S-1}$

Therefore $X(S) = \frac{2}{S^2(S-1)} - \frac{2}{S^3(S-1)} - \frac{1}{S(S-1)} + \frac{1}{S-1}$, thus $x(t) = t^2 + 1$

II) Consider; $y(x) = \sum_{n=0}^{\infty} a_n (x - x_0)^n$, $y'(x) = \sum_{n=1}^{\infty} n\, a_n (x - x_0)^{n-1}$,

$$y''(x) = \sum_{n=2}^{\infty} n(n-1)\, a_n (x - x_0)^{n-2}$$

Substitute in the above D.E., we get

$$\sum_{n=2}^{\infty} n(n-1)\, a_n (x-x_0)^{n-2} - x\sum_{n=1}^{\infty} n\, a_n (x-x_0)^{n-1} + \sum_{n=0}^{\infty} a_n (x-x_0)^n = \sum_{n=2}^{\infty} n(n-1)\, a_n (x-x_0)^{n-2} -$$

$$\sum_{n=1}^{\infty} n\, a_n (x-x_0)^n - x_0 \sum_{n=1}^{\infty} n\, a_n (x-x_0)^{n-1} + \sum_{n=0}^{\infty} a_n (x-x_0)^n = 0.$$

Put n = s+2 for 1st term, n = s for the 2nd term, n = s+1 for 3rd term and n = s for 4th term, we get:

$$\sum_{s=0}^{\infty} (s+2)(s+1)\, a_{s+2} (x-x_0)^s - \sum_{s=1}^{\infty} s\, a_s (x-x_0)^s \quad x_0 \sum_{s=0}^{\infty} (s+1)\, a_{s+1} (x-x_0)^s + \sum_{s=0}^{\infty} a_s (x-x_0)^s = 0,$$

thus $2a_2 - x_0 a_1 + a_0 + \sum_{s=1}^{\infty} ((s+2)(s+1)\, a_{s+2} - x_0 (s+1)\, a_{s+1} - (s-1)\, a_s)(x-x_0)^s = 0.$

By comparing of coefficients, we get:

$2a_2 - x_0 a_1 + a_0 = 0$, from which $a_2 = \frac{x_0 a_1 - a_0}{2}$ and by comparing coefficients of $(x - x_0)^s$, s = 1, 2, 3, ….,we get

$$a_{s+2} = \frac{x_0(s+1)a_{s+1} + (s-1)a_s}{(s+2)(s+1)}$$

Therefore

$a_3 = \frac{x_0 a_2}{3} = \frac{x_0(x_0 a_1 - a_0)}{6}$, $a_4 = \frac{3x_0 a_3 + a_2}{12} = \frac{(x_0^2+1)(x_0 a_1 - a_0)}{24}$,

The solution will be in the form

$$y(x) = \sum_{n=0}^{\infty} a_n (x - x_0)^n = A\left(1 - \frac{1}{2}(x - x_0)^2 - \frac{x_0}{6}(x - x_0)^3 - \frac{(x_0^2+1)}{24}(x - x_0)^4 + \ldots\right) +$$
$$B\left((x - x_0) + \frac{x_0}{2}(x - x_0)^2 + \frac{x_0^2}{6}(x - x_0)^3 + \frac{(x_0^2+1)x_0}{24}(x - x_0)^4 + \ldots\right),$$ where $a_0 = A$, $a_1 = B$,

III) Let the straight line is y = ax+b , we have to

i	x_i	x_i^2	y_i	$x_i y_i$
1	-1	1	3	-3
2	1	1	7	7
3	3	9	2	6
Sum	$\sum_{i=1}^{3} x_i = 3$	$\sum_{i=1}^{3} x_i^2 = 11$	$\sum_{i=1}^{3} y_i = 12$	$\sum_{i=1}^{3} x_i y_i = 10$

Therefore $a = \frac{3(10) - 3(12)}{3(11) - 9} = -1/4$, $b = \frac{12(11) - 10(3)}{3(11) - 9} = 17/4$