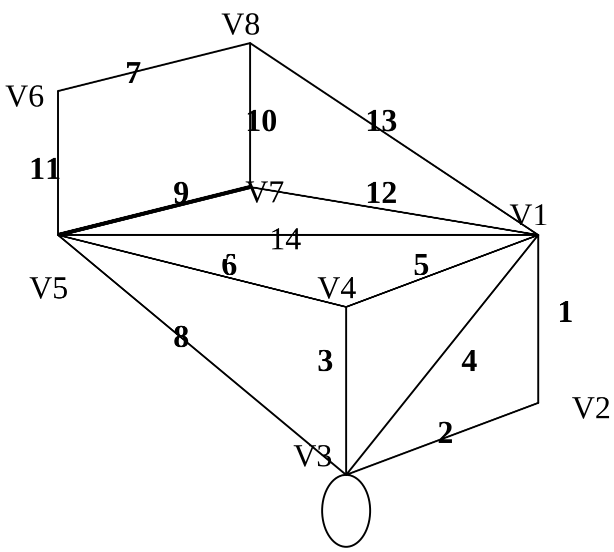
Benha University
Faculty of Engineering- Shoubra
(**Electrical**)Engineering Department
Power

Final term exam Date: **18-5-2014**
Mathematics **3B** Code: **EMP 272**
Duration : **3 hours**

Answer the following questions No. of questions : **3** Total Mark: 80

Question1 **30 Marks**



I) Define Cycle – Length of walk – Degree of vertices – Tree – Condition of existence for Eulerian path and Eulerian circuit – Sub graph

II) Referring to the given graph, find

a) Cycle of length 6 starts and ends at V1

b) Degree of the graph c) Spanning tree

d) Eulerian path e) Regular sub graph

f) Adjacency matrix

Question 2 **20 Marks**

Find U(x, y) using finite difference method for P.D.E. $U_{xx} + U_{yy} = 4$. If we consider a square mesh of height consists of 4 equal parts with B.C. $U(0,y) = y^2$, $U(1,y) = (y-1)^2$, $0 \le y \le 2$ & $U(x,0) = x^2$, $U(x,2) = (x-2)^2$, $0 \le x \le 1$. Solve the constructed system Gauss Jordan Method.

Question 3 **30 Marks**

I) Solve the following system of equations using Picard up to 2^{nd} approximation

$$x` - 3y` = -2t + x - 2y - 7, \quad 2x` + y` = 10t + y + 3 - t^2, \; x(0) = 1, \; y(0) = -3$$

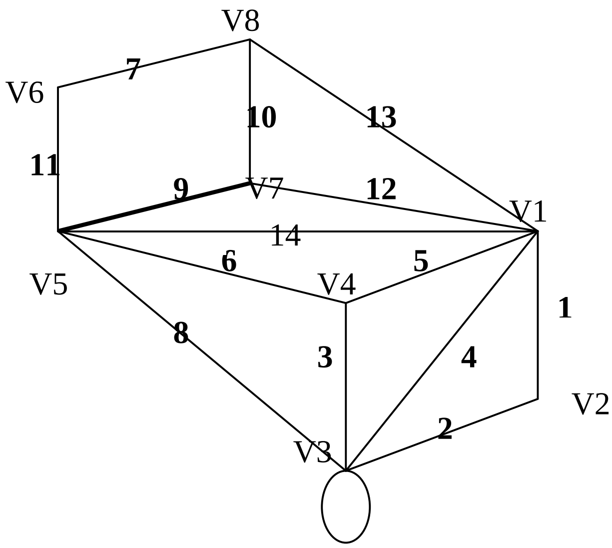
Find x(0.2) by taking h = 0.1 using Euler method.

II) Find the constants of the following curve $y(x) = \dfrac{1}{a + bx + cx^2}$ that fit the following data (-1,2), (3,4), (6,9).

Board of examiners : Dr. eng. Khaled El Naggar

Model answer

Answer of Question1



a) Define

Cycle: **Closed walk with no repeated vertices except that the initial vertex is the terminal vertex**

Length of walk: **is the number of edges in the walk.**

Degree of vertices: **The degree deg(v) of vertex v is the number of edges incident on v**

Tree: **is a connected graph which has no cycles**

Euler Path existence**: 2 or no odd-degree nodes exist in the graph.**

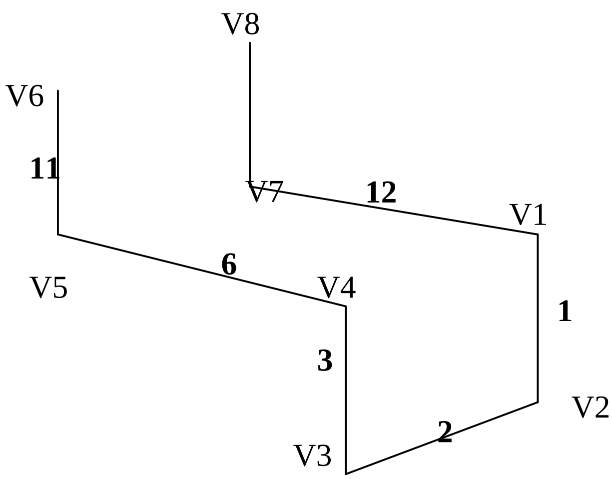
Euler Circuit existence**: no odd-degree nodes exist in the graph.**

Sub graph: **consists of group of vertices and edges which belong to the main graph**

a) Cycle of length 6 starts and ends at V1 : V1 V4 V5 V6 V8 V7 V1

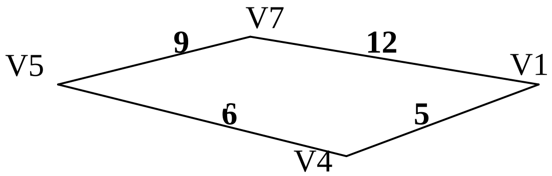
b) Degree of the graph : 6 + 2 +6 +3+5+2+3+3 =30

c) The Spanning tree



d) No Eulerian path

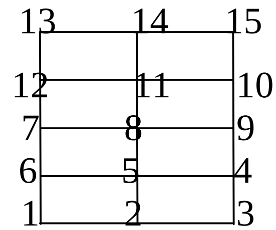
e) Regular sub graph



f) Adjacency matrix is

$$\begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

Answer of Question 2



h = k = 0.5, therefore

$u_1 = 0$, $u_2 = 0.25$, $u_3 = 1$, $u_6 = 0.25$, $u_7 = 1$, $u_{12} = 2.25$, $u_{13} = 4$, $u_{14} = 2.25$, $u_{15} = 1$, $u_{10} = 0.25$, $u_9 = 0$, $u_4 = 0.25$ and the general elliptic equation will be

$$u_{i+1,j} + u_{i-1,j} + u_{i,j+1} + u_{i,j-1} - 4u_{i,j} = 4h^2$$

Thus $u_6 + u_4 + u_8 + u_2 - 4u_5 = 0.25(4) \Rightarrow u_8 - 4u_5 = 0.25$

$u_7 + u_9 + u_5 + u_{11} - 4u_8 = 0.25(4) \Rightarrow \quad u_5 + u_{11} - 4u_8 = 0$

$u_{12} + u_{10} + u_8 + u_{14} - 4u_{11} = 0.25(4) \Rightarrow u_8 - 4u_{11} = -1.75$

By solving the 3 equations, we can get u_5, u_8, u_{11}

Answer of Question 3

By solving the 2 equations, we get $x' = \frac{1}{7}[28t + y + x - 3t^2 + 2]$, $y' = \frac{1}{7}[14t + 5y - 2x - t^2 + 17]$,

$x_0 = 1$, $y_0 = -3$, $t_0 = 0$, therefore $x_{n+1}(t) = x_0 + \frac{1}{7}\int_0^t [28t + y_n + x_n - 3t^2 + 2]\,dt$,

Thus $x_1(t) = x_0 + \frac{1}{7}\int_0^t [28t + y_0 + x_0 - 3t^2 + 2]\,dt = 1 + \frac{1}{7}\int_0^t [28t - 3t^2]\,dt = 1 + 2t^2 - \frac{t^3}{7}$

And $y_{n+1}(t) = y_0 + \frac{1}{7}\int_0^t [14t + 5y_n - 2x_n - t^2 + 17]\,dt$

Thus $y_1(t) = y_0 + \frac{1}{7}\int_0^t [14t + 5y_0 - 2x_0 - t^2 + 17]\,dt = -3 + \frac{1}{7}\int_0^t [14t - t^2]\,dt = -3 + t^2 - \frac{t^3}{21}$

II) Let $Y = 1/y = a + bx + cx^2$, so that the constants can be obtained using the following matrix form :

$$\begin{pmatrix} N & \sum_{i=1}^{N} x_i & \sum_{i=1}^{N} x_i^2 \\ \sum_{i=1}^{N} x_i & \sum_{i=1}^{N} x_i^2 & \sum_{i=1}^{N} x_i^3 \\ \sum_{i=1}^{N} x_i^2 & \sum_{i=1}^{N} x_i^3 & \sum_{i=1}^{N} x_i^4 \end{pmatrix} \begin{pmatrix} c \\ b \\ a \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^{N} Y_i \\ \sum_{i=1}^{N} x_i Y_i \\ \sum_{i=1}^{N} x_i^2 Y_i \end{pmatrix}$$

Given N= 3 , $\sum_{i=1}^{N} x_i = 8$, $\sum_{i=1}^{N} x_i^2 = 46$, $\sum_{i=1}^{N} x_i^3 = 242$, $\sum_{i=1}^{N} x_i^4 = 1378$, $\sum_{i=1}^{N} Y_i = 0.861$, $\sum_{i=1}^{N} x_i Y_i = 0.92$, $\sum_{i=1}^{N} x_i^2 Y_i = 6.75$, thus by Cramer rule we get a , b and *c* such that: $a = \frac{\Delta_a}{\Delta}$, $b = \frac{\Delta_b}{\Delta}$, $c = \frac{\Delta_c}{\Delta}$,

where $\Delta = \begin{vmatrix} 3 & 8 & 46 \\ 8 & 46 & 242 \\ 46 & 242 & 1378 \end{vmatrix}$, $\Delta_a = \begin{vmatrix} 0.86 & 8 & 46 \\ 0.92 & 46 & 242 \\ 6.75 & 242 & 1378 \end{vmatrix}$, $\Delta_b = \begin{vmatrix} 3 & 0.86 & 46 \\ 8 & 0.92 & 242 \\ 46 & 6.75 & 1378 \end{vmatrix}$,

$$\Delta_c = \begin{vmatrix} 3 & 8 & 0.86 \\ 8 & 46 & 0.92 \\ 46 & 242 & 6.75 \end{vmatrix}$$