Benha University Faculty of Engineering- Shoubra Mechanical Engineering Department 1st Year Mechanical power		Final Term Exam Date: 31st of May 2015 Mathematics (2-B) - EMP 133 Duration : 3 hours

Answer all the following questions	• No. of questions: 8
The exam. Consists of one page	• Total Mark: 100 Marks

1) Evaluate the following integrals **[20]**

a) $\int_a^b (x-a)^m (b-x)^n \, dx$ **b)** $\int_0^\infty \frac{x^n \, dx}{a^x}$ **c)** $\int_{-\pi/4}^{\pi/4} [\sqrt{2}\cos(\theta+\pi/4)]^{1/3} d\theta$ **d)** $\int_0^\infty (\frac{e^{2t} - \cos 3t}{t}) e^{-3t} \, dt$

2) Solve the following differential equations using Laplace Transform: **[10]**

a) $\frac{dx}{dt} + x + 4y = 10, \quad x - \frac{dy}{dt} - y = 0$, $x(0) = 4$, $y(0) = 3$

b) $y'' + y' = 2(1 + t)$, $y(2) = 4$, $y'(2) = 4$

3) Find Laplace transform for $f(t) = t \sin 2t \cosh 3t + \begin{cases} 6t & 0 < t \le 2 \\ 3t^2 & 2 < t \le 3 \\ 27 & t > 3 \end{cases} + \frac{\cos 2t - \cos 3t}{t}$ **[10]**

4) Find inverse Laplace for $F(s) = \frac{s}{s^2 + 4s + 9} + \frac{e^{-2s}}{s+3} + \frac{1}{(s+9)^3}$ **[10]**

5) A certain deck of cards contains 3 blue cards, 5red cards, and 2 green cards. If two cards are randomly drawn from the deck, what is the probability that at least one is blue? **[10]**

If X is the random variable where X is the number of red cards, find expected value, variance, cumulative density function , mode , Var(3X-1), and sketch P.d.f. **[10]**

6) Let the r.v. X be the distance in feet between bad records on a used computer tape. Suppose that a reasonable probability model for X is given by the p.d.f. $f(x) = \frac{1}{40} e^{-x/40}$, $x > 0$

Find m.g.f. and the mean, standard deviation. **[10]**

7) Suppose the probability that a college freshman will graduate is 0.6. Three sisters (triplets) enter college at the same time. What is the probability that at most 2 sisters will graduate? **[10]**

8) Two fair dice are rolled. Find the probability that the sum rolled is at least ten, given:

A) At least one die comes up six B) The same number appears on both dice **[10]**

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Model answer

Answer of question 1

a) put x-a = y $\Rightarrow$ dx = dy, therefore

$$\int_a^b (x-a)^m (b-x)^n dx = \int_0^{b-a} y^m [(b-a)-y]^n dy = (b-a)^n \int_0^{b-a} y^m [1-\frac{y}{b-a}]^n dy .$$

put z = $\frac{y}{b-a}$ $\Rightarrow$ (b-a) dz = dy, thus

$$\int_a^b (x-a)^m (b-x)^n dx = (b-a)^{m+n+1} \int_0^1 z^m [1-z]^n dz = (b-a)^{m+n+1} \beta(m+1, n+1)$$

b) Since x lna = y,

therefore $\int_0^\infty \frac{x^n dx}{a^x} = \int_0^\infty x^n a^{-x} dx = \int_0^\infty x^n e^{-x \ln a} dx = \int_0^\infty (\frac{y}{\ln a})^n e^{-y} \frac{dy}{\ln a} =$

$$\frac{1}{[\ln(a)]^{n+1}} \int_0^\infty y^n e^{-y} dy = \frac{1}{[\ln(a)]^{n+1}} \Gamma(n+1)$$

c) put $\phi = \theta + \frac{\pi}{4} \Rightarrow d\phi = d\theta$, therefore

$$\int_{-\pi/4}^{\pi/4} [\sqrt{2}\cos(\theta + \pi/4)]^{1/3} d\theta = \int_0^{\pi/2} [\sqrt{2}\cos\phi]^{1/3} d\phi = \frac{2^{1/6}}{2} \beta(2/3, 1/2)$$

d) Since $L\{\frac{e^{2t} - \cos 3t}{t}\} = \int_s^\infty [\frac{1}{s-2} - \frac{s}{s^2+9}] ds = Ln \frac{\sqrt{s^2+9}}{s-2}$, therefore

$$\int_0^\infty (\frac{e^{2t} - \cos 3t}{t}) e^{-3t} dt = Ln \frac{\sqrt{s^2+9}}{s-2} \Bigg|_{s=3} = Ln\sqrt{18}$$

Answer of question 2

a) Take Laplace to the above equations such that:

s X(s) - x(0) + X(s) + 4 Y(s) = 10/s, X(s) – [s Y(s) - y(0)] - Y(s) = 0, thus (s +1) X(s) + 4 Y(s) = 10/s + 4…(1) X(s) – (s +1)Y(s) = -3…(2)

Multiply equation (2) by (s+1) and subtract, therefore $[4+(s+1)^2]$ Y(s) = 10/s + 3s+7

Thus $Y(s) = \frac{10}{s[4+(s+1)^2]} + \frac{3(s+1)+4}{[4+(s+1)^2]} = \frac{2}{s} + \frac{(s+1)+2}{[4+(s+1)^2]}$

Hence $y(t) = 2 + e^{-t}(\cos 2t + \sin 2t)$, but $x = \frac{dy}{dt} + y$, therefore

$$x = 2+2e^{-t}(\cos 2t - \sin 2t)$$

b) By taking Laplace to both sides, we get

$S^2 Y(S) - Sy(0) - y`(0) + SY(S) - y(0) = 2[\frac{1}{s} + \frac{1}{s^2}]$.

Let $y(0) = a$ and $y`(0) = b$, therefore $(S^2 + S)Y(S) = 2[\frac{1}{s} + \frac{1}{s^2}] + (S+1)a + b \Rightarrow$

$Y(S) = 2[\frac{1}{s^2(s+1)} + \frac{1}{s^3(s+1)}] + \frac{1}{s} a + \frac{1}{s(s+1)} b \Rightarrow y(t) = 2[t + e^{-t} - 1] + 2[\frac{t^2}{2} - e^{-t} - t + 1]$

$+ a + b[1 - e^{-t}]$, but $y(2) = 4$ and $y`(2) = 4$, therefore $a = b = 0$. Hence $y(t) = t^2$.

Answer of question 3

$L\{t\sin 2t\cosh 3t\} = L\{t\sin 2t(e^{3t} + e^{-3t})/2\} = \frac{2(s-3)}{[(s-3)^2+4]^2} + \frac{2(s+3)}{[(s+3)^2+4]^2}$

Since $g(t) = \begin{cases} 6t & 0 < t \le 2 \\ 3t^2 & 2 < t \le 3 \\ 27 & t > 3 \end{cases}$ $= 6t[u(t) - u(t-2)] + 3t^2[u(t-2) - u(t-3)] + 27u(t-3)$, therefore

$g(t) = 6t\,u(t) - 6(t-2+2)u(t-2) + 3(t-2+2)^2 u(t-2) - 3(t-3+3)^2 u(t-3) + 27u(t-3) =$

$6t\,u(t) + 6(t-2)u(t-2) + 3(t-2)^2 u(t-2) - 3(t-3)^2 u(t-3) - 18(t-3)u(t-3)$, thus

$G(s) = 6/s + (6/s^2)e^{-2s} + (6/s^3)e^{-2s} + (6/s^3)e^{-3s} - (18/s^2)e^{-3s}$

$L\{\frac{\cos 2t - \cos 3t}{t}\} = \int_s^{\infty}(\frac{s}{s^2+4} - \frac{s}{s^2+9})ds = \frac{1}{2}\ln[\frac{s^2+9}{s^2+4}]$

Answer of question 4

a) Since $\frac{s+2-2}{(s+2)^2+5} = \frac{s+2}{(s+2)^2+5} - \frac{2}{(s+2)^2+5}$, therefore

$$f(t) = e^{-2t}(\cos\sqrt{5}\,t - \frac{2}{\sqrt{5}}\sin\sqrt{5}\,t) + e^{-3(t-2)}U(t-2) + \frac{1}{2!}t^2 e^{-9t}$$

Answer of question 5

$P(B \ge 1) = P(B = 1) + P(B = 2) = 2P(GB) + 2P(RB) + P(BB) = 7/15 + 1/15 = 8/15$

Let r.v. X is the red cards, then $P(X=0) = P(GG) + P(BB) + 2P(BG) = 2/9$,

P(X=1) =2 P(RG) + 2P(RB) = 5/9, P(X=2) = P(RR) = 2/9, E(X) = 1, Var(X) = 4/9, mode = {1}, V(3X-1) = 9 V(X) = 4 and C.d.F. is expressed as

X	0	1	2
F(X)	2/9	7/9	1

Answer of question 6

$P(X > x) = 1 - P(X<x) = 1-\int_0^x \frac{1}{40} e^{-x/40} dx = e^{-x/40}$. To get the median a such that P(X< a) = 0.5,

therefore $\int_0^a \frac{1}{40} e^{-x/40} dx = 0.5$, thus $1- e^{-a/40} = 0.5 \Rightarrow$ a = -40 ln(0.5) = 27.726, and

m.g.f. = $\int_0^\infty e^{tx} (\frac{1}{40} e^{-x/40}) dx = \int_0^\infty \frac{1}{40} e^{\frac{-(1-40t)x}{40}} dx = \frac{1}{1-40t} = \phi(t)$, therefore $\mu_1' = \phi'(0) =$

$\left.\frac{40}{(1-40t)^2}\right|_{t=0}$ = 40 = E(X) and $\mu_2' = \phi''(0) = \left.\frac{3200}{(1-40t)^3}\right|_{t=0}$ = 3200 = $E(X^2)$, hence var(X) =

$E(X^2) - [E(X)]^2$ = 3200 – 1600 = 1600, so standard deviation = 40

Answer of question 7

P = 0.6, n = 3, $P(X \le 2) = \sum_{x=0}^{2} {}^3c_x (0.6)^x (0.4)^{3-x} = 1 - P(X=3) = 1 - {}^3c_3 (0.6)^3 (0.4)^0 = 0.784$

Answer of question 8

A- P{at least ten/ At least one die comes up six} = 5/11, since the event ‘ At least one die comes up six’ can be expressed by: { (6,1), (6,2), (6,3), <u>(6,4), (6,5), (6,6)</u>, (1,6), (2,6), (3,6), <u>(4,6), (5,6)</u>}

B- P{at least ten/ The same number appears on both dice} = 2/6, since the event ‘The same number appears on both dice’ can be expressed by: { (1,1), (2,2), (3,3), (4,4), <u>(5,5), (6,6)</u> }