Benha University Faculty of Engineering- Shoubra (Civil)Engineering Department		Final term exam Date: 1 -6- 2015 Mathematics & Statistics Code: EMP 151 Duration : 3 hours
Answer all the following questions	No. of questions : 5	Total Mark: 70 marks

Question 1 **[20 Marks]**

a) Test the series i) $\sum_{n=1}^{\infty} \frac{(\ln n)^3}{n}$ ii) $\sum_{n=1}^{\infty} \frac{1}{n^2+3n+10}$ iii) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} n}{5^n}$

b) Find interval of convergence for the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^{2n-1}}{(2n-1)!}$

c) Expand $f(x,y) = e^x \cos y$ **in powers of x and y up to third degree approximation**

d) If W(x,y) = ln(x^2 y), prove that $x^2 w_{xx} + 2xy\, w_{xy} + y^2 w_{yy} + 3 = 0$

e) Use Lagrange Multiplier to find the extrema of the function:

$$f(x,y) = 2x^2 + xy - y^2 + y \quad \text{subject to} \quad 2x + 3y = 1$$

Question 2 **[20 Marks]**

Solve the following differential equations:

i) $(2y - x)\, y` + 2y = x + 1$ ii) $x^2 y^2 y` + x y^3 = 1$ iii) $x y` + y(1 - x y \sin x) = 0$

iv) $y``` - y`` - 6 y` = x^2 + 1$ v) $y`` - 4 y` + 4 y = x^2 e^{2x} \sin(2x)$

Question 3 **[15 Marks]**

I) If **$f(x) = a x + b x^3$** is a discrete probability distribution, $0 < x \leq 3$, given E(3X-1) = 6.5 , **find Law of distribution, V(3X- 7), mode and median.**

II) A pair of fair dice is thrown. If two numbers are different, **find the probability** that:

a) The sum is 6 b) an ace appears c) the sum is 4 or less d) the sum is even

Question 4 **[15 Marks]**

I) Consider the function $f(x) = c(2x - x^2)$ $0 < x < 3/2$. Could f be a probability density function? If **so determine c, mode and median**

II) Six different colored dice are rolled, the random variable is the numbers of dice that show a "4", find the probability that at least 3 dice show a "4." **Find expected value and standard deviation.**

Board of Examiners: Dr. Ibrahim Sakr & Dr. eng. Khaled el Naggar

Answer of Question 3

I) Since $f(x) = a\,x + b\,x^3$ is a discrete probability distribution, therefore $6a + 36b = 1$. But $E(3X-1) = 6.5$, thus $E(X) = 2.5$ and hence $14a + 98b = 2.5$. By solving the 2 equations, we get $a = 0.1$ and $b = 0.011$, therefore the law of distribution is

x	1	2	3
f(x)	0.111	0.288	0.597

$E(x^2) = 6.636$, hence $Var(X) = E(x^2) - [E(x)]^2 = 0.386 \Rightarrow V(3X-7) = 3.474$

cumulative density function is

x	1	2	3
f(x)	0.111	0.499	1

Median = {2}, Mode = { 3}

II) A = { two numbers are different} and B = {Sum is 6}, P(B/A) = 4/30 = 2/15,

C = { an ace appear}, P(C/A) = 10/30 = 1/3,

D = { the sum is 4 or less}, P(D/A) = 4/30 = 2/15,

E = { the sum is even } = 12/30 = 6/15.

Answer of Question 4

I) If $2x - x^2 > 0$, then $x \in [0,2]$ and hence $f = c(2x - x^2)$ is a probability density function, where

$$\int_0^{3/2} c(2x - x^2)dx = 1 \Rightarrow c(x^2 - \frac{x^3}{3})_{x=0}^{3/2} = 1 \Rightarrow c = 8/9 \text{ and mode} = \{1\}$$

Since cumulative density function is expressed by $F(x) = \int_0^x \frac{8}{9}(2x - x^2)dx = \frac{8}{9}(x^2 - \frac{x^3}{3})$, thus

$\int_0^x \frac{8}{9}(2x - x^2)dx = \frac{8}{9}(x^2 - \frac{x^3}{3}) = 0.5$, from which x is the median

II)Since n = 6 and p = 1/6, thus q = 5/6, hence $p(x \geq 3) = \sum_{x=3}^{6} {}^{6}c_x (\frac{1}{6})^x (\frac{5}{6})^{6-x}$, E(X) = np = 1

and Var(X) = npq = $5/6 \Rightarrow \sigma_x = \sqrt{5/6}$.