Benha University
Faculty of Engineering- Shoubra
ESE Program
Final Examination (تخلفات)

Subject: Mathematics 2
Code: EMP 102
Date: 23-5- 2016
Duration : 3 hours

- **Answer all the following questions**
- **Illustrate your answers with sketches when necessary.**
- **The Exam. Consists of one Pages**
- **No. of Questions: 2**
- **Total Mark:** 40 Marks

Question (1):

a- If A = (1 , 5), B = (-3 , 2), $C \in \overleftrightarrow{AB}$ such that 4AC = 7CB, find coordinates of C which divides $\overline{AB}$ internally. **[4]**

b- Find p and q such that $2x^2 + pxy + 2y^2 + x + qy - 1 = 0$ represent pair of parallel straight lines, and find the distance between the two lines. **[6]**

c- Find the equation of the circle which touch the straight line 6x –8y =0 and whose center is (3, 1).Then find the point of contact. **[6]**

d) Find k such that the 2 circles $x^2 + y^2 + 2x + ky - 2 = 0$ & $x^2 + y^2 + 6x + 4y + k = 0$ are orthogonal. **[4]**

Question (2)

a) Evaluate the following integral **[8]**

i) $\int \sqrt{\frac{1+x}{1-x}}\,dx$, ii) $\int [\cos 2x \sin 5x]\,dx$, iii) $\int x\sqrt{1+\sqrt{x}}\,dx$, iv) $\int x^{n-1} e^{x^n}\,dx$

b) Find the area bounded by the curves $y = x^2$ and $y = \sqrt{x}$ **[4]**

c) Find the length of the curve $y = x^2$ from x = 0 to x = 1 **[4]**

d) Solve the D.E. $y' = \tan^3 x + x \cos x$, $y(0) = 1$ **[4]**

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Answer of question 1

a) $\because \frac{AC}{CB} = \frac{7}{4}$, therefore $m_1 = 7$, $m_2 = 4$, and A = (1 , 5), B = (-3 , 2).

Let C = (x_3 , y_3), such that $x_3 = \frac{1(4) - 3(7)}{7+4} = \frac{-17}{11}$ & $y_3 = \frac{5(4) + 2\,(7)}{7+4} = \frac{34}{11}$, thus C = ($\frac{-17}{11}$, $\frac{34}{11}$).

b) Since a = 2, h = p/2, b = 2, g = ½, f = q/2, c = -1 and since the two lines are parallel, therefore $h^2 = ab = 4 \Rightarrow p^2/4 = 4 \Rightarrow p = 4$ or p = -4 .

At p = -4, the above equation represents pair of straight lines if

$$\begin{vmatrix} 2 & -2 & 1/2 \\ -2 & 2 & q/2 \\ 1/2 & q/2 & -1 \end{vmatrix} = 0 \Rightarrow 2[-2-q^2/4]+2[2 - q/4] - [q + 1]/2 = 0 \Rightarrow [q + 1]^2 = 0 \Rightarrow$$

p = -1, therefore the equation of the pair of st. lines is $2x^2-4xy+2y^2 +x-y-1=0 \Rightarrow$ $(x-y+c_1)(2x-2y+c_2) =0$, therefore $2x^2 - 4xy + 2y^2 + (c_2 +2c_1)x -(2c_1+ c_2)y + c_1c_2 = 0$ and by comparing coefficients of x and the constant such that:

$$c_2 +2c_1 = 1, \qquad c_1c_2= -1$$

By solving the two equations simultaneously, we get $c_1 = 1$ and $c_2 = -1$ and therefore the two lines are $x - y+ 1 = 0$ and $2x - 2y - 1= 0$

If the unknown is the coefficient of x or y, then we can get the unknown coefficient by separating the two lines and by comparing the coefficients, we can get the unknown coefficient. For the above example, we can get the two parallel lines separately as follows:

$2x^2 - 4xy + 2y^2 + (c_2 +2c_1)x -(2c_1+ c_2)y + c_1c_2 = 0$, therefore and by comparing coefficients of x and the constant such that: $c_2 +2c_1 = 1, \quad c_1c_2= -1$

By solving the two equations simultaneously, we get $c_1 = 1$ and $c_2 = -1$ and therefore the two lines are $L_1 : x - y+ 1 = 0$ and $L_2: 2x - 2y - 1= 0$, but $q = -(2c_1+ c_2) = -1$.

To get the shortest distance between the two lines, put x = 0 in L_1, therefore A(0, 1) satisfy L_1 and drop a perpendicular line from (0,1) on L_2 at point B, so that

$$AB = \frac{|-3|}{\sqrt{2^2 + 2^2}} = \frac{3}{2\sqrt{2}}.$$

c) The line joining the center and the point of contact is perpendicular to the tangent and equal the raduis of the circle, hence the raduis of the cirle is 1 and therefore the equation of the circle is $(x-3)^2 + (y-1)^2 = 1 \Rightarrow x^2 + y^2 - 6x - 2y + 9 = 0$ and the point of contact is (12/5, 9/5)

d) Since the 2 circles are othogonal, therefore $2g_1 g_2 + 2f_1 f_2 = c_1 + c_2$

$\Rightarrow 2(1)(3) + 2(k/2)(2) = -2 + k \Rightarrow k = -8$

Answer of question 2

i) $\int \sqrt{\frac{1+x}{1-x}}dx = \int \frac{1+x}{\sqrt{1-x^2}}dx = \int \frac{dx}{\sqrt{1-x^2}} + \int \frac{xdx}{\sqrt{1-x^2}} = \sin^{-1}x - \sqrt{1-x^2} + c$

ii) $\int [\cos 2x \sin 5x]\,dx = \frac{1}{2}\int [\sin 3x + \sin 7x]dx = \frac{1}{2}[\frac{-\cos 3x}{3} + \frac{-\cos 7x}{7}]$

iii) Put $\sqrt{1+\sqrt{x}} = u \Rightarrow \sqrt{x} = u^2 - 1 \Rightarrow x = u^4 - 2u^2 + 1 \Rightarrow dx = 4[u^3 - u]\,du$,

therefore $\int x\sqrt{1+\sqrt{x}}\,dx = 4\int u[u^4 - 2u^2 + 1][u^3 - u]du$

$= 4\int [u^8 - 3u^6 + 3u^4 - u^2]du = 4[\frac{u^9}{9} - 3\frac{u^7}{7} + 3\frac{u^5}{5} - \frac{u^3}{3}]$

$= 4\sqrt{1+\sqrt{x}}^{\,3}[\frac{\sqrt{1+\sqrt{x}}^{\,6}}{9} - 3\frac{\sqrt{1+\sqrt{x}}^{\,4}}{7} + 3\frac{\sqrt{1+\sqrt{x}}^{\,2}}{5} - \frac{1}{3}] + c$

iv) $\int x^{n-1}e^{x^n}\,dx = \frac{1}{n}\int nx^{n-1}e^{x^n}\,dx = \frac{e^{x^n}}{n} + c$

b) Area bounded between the 2 cuves such that $\int [\sqrt{x} - x^2]dx = \left.\frac{2x^{3/2} - x^3}{3}\right|_0^1 = \frac{1}{3}$

c) $L = \int_0^1 \sqrt{1 + y'^2}\,dx = \int_0^1 \sqrt{1+4x^2}\,dx$

put $2x = \tan\theta$, therefore $2dx = \sec^2\theta\, d\theta$, therefore

$$L = \int_0^1 \sqrt{1+4x^2}\, dx = \frac{1}{2}\int \sqrt{1+\tan^2\theta}\, \sec^2\theta d\theta$$

$$= \frac{1}{2}\int \sec^3\theta\, d\theta = \frac{\ln\left|\sec\theta + \tan\theta\right| + \sec\theta\tan\theta}{4} = \frac{\ln[\sqrt{4x^2+1} + 2x] + 2x\sqrt{4x^2+1}}{4}$$

$$= \frac{\ln[\sqrt{5}+2] + 2\sqrt{5}}{4}$$ unit length

d) $y = \int \tan x(\sec^2 x - 1)dx + x(\sin x) - (-\cos x) = \frac{\tan^2 x}{2} + \ln\left|\cos x\right| + x\sin x + \cos x + c$

But y(0) = 1, therefore c = 0

- Intended Learning Outcomes of Course (ILO's)

a. **Knowledge and Understanding Skills:** On completing this course, students will be able to demonstrate the knowledge and understanding of :
   a.1) Recognize the concepts and theories of analytical geometry and integration related to the energy and sustainable energy engineering studies.(A1)
   a.2) Recognize the methodologies of solving integration problems. (A5)

b. **Intellectual Skills:** At the end of this course, the students will be able to:

   b.1) Select appropriate integration, conic sections for modelling and analysing energy problems.(B1)

c. **Practical and Professional Skills:** On completing this course, the students are expected to be able to:
   c.1) Apply knowledge of definite integration and conic sections to solve energy and sustainable energy engineering problems.(C1)
   c.2) Apply numerical modelling methods to solve energy engineering problems. (C7)

d. **General and Transferable Skills:** At the end of this course, the students will be able to:
   d1) Communicate effectively (D3)

Questions	Total marks	Achieved ILOS
Q1	20	a1,b1
Q2	20	a1,a2, b1,c1