Benha University
Faculty of Engineering- Shoubra
Industrial Engineering Program
Final Examination

Subject: Mathematics 4
Code: EMP 202
Date: 28/5/2016
Duration : 3 hours

- **Answer all the following questions**
- **Illustrate your answers with sketches when necessary.**
- **The Exam. Consists of one Page**
- **No. of Questions: 2**
- **Total Mark:** 40 Marks

Question 1 [20 Marks]

Solve the following differential equations

a) $(x+2y)\,dx - (2x-y+1)\,dy = 0$ b) $y`` + y = \tan(x)$ c) $(2y+x^2+1)\dfrac{dy}{dx} - 9x^2 + 2xy = 0$

d) $y'' + 5y' + 4y = e^{5x}\cos 2x$ e) $xy` + y = -x^3$

Question 2 [10 Marks]

Solve using Picard's method up to 2^{nd} approximation the following differential equation:
$y`` = x^3\,(y` + y)$ , $y(0) = 1$ , $y`(0) = ½$, then find $y(0.2)$ using Euler method, $h = 0.1$.

Question 3 [10 Marks]

a) Find inverse Laplace for the function $F(s) = \dfrac{s\,e^{-2s}}{s^2+6s+20} + \dfrac{1}{(3s+4)^3}$

b) Find Laplace transform for the function $f(t) = t\sin 2t\cosh 3t + e^{-3t}\int_{u=0}^{t} e^{-2u}\sin u\,du$

Dr. Eng. Khaled El Naggar

Model answer

1-a) Put x = X-2/5, y = Y+1/5, hence (X+2Y) dX + (2X-Y) dY = 0

Then, put Y = v X, thus dY = v dX + X dv

Therefore (X+2vX) dX - (2X-vX) (v dX + X dv) = 0

Hence $(X + v^2X)\, dX - (2X-vX)\, X\, dv = 0 \Rightarrow \frac{dx}{x} + \frac{v-2}{v^2+1}dv = 0$

$$\Rightarrow \ln X + \frac{1}{2}\ln(v^2+1) - 2\tan^{-1} v \;\Rightarrow \ln(x+\frac{2}{5}) + \frac{1}{2}\ln((\frac{y-1/5}{x+2/5})^2+1) - 2\tan^{-1}(\frac{y-1/5}{x+2/5})$$

1-b) The characteristic equation is $m^2+1=0 \Rightarrow$ m = $\pm i$ =, thus y_c = (c_1 cosx + c_2 sinx) , and $y_1(x)$ = cosx , $y_2(x)$ = sinx , therefore $W(y_1, y_2) = y_2'y_1 - y_2y_1' = 1$, g(x) = tanx

$$\int \frac{y_2g(x)}{W(y_1,y_2)}dx = \int \frac{\text{sinx tanx}}{1}dx = \int \frac{\sin^2 x}{\cos x}dx = \int \frac{1-\cos^2 x}{\cos x}dx = \ln(\sec x + \tan x) - \sin x$$

$$\int \frac{y_1g(x)}{W(y_1,y_2)}dx = \int \frac{\text{cosx(tanx)}}{1}dx = \int \text{sinx}dx = -\cos x$$

But $Y_P(x) = -y_1\int \frac{y_2g(x)}{W(y_1,y_2)}dx + y_2\int \frac{y_1g(x)}{W(y_1,y_2)}dx$, thus Y = $Y_c + Y_p$

1-c) $(-9x^2+2xy)dx + (2y+x^2+1)dy = 0 \Rightarrow \mathbf{M_y = N_x}$ **,** therefore the D.E. is exact.

Let f(x,y) = c $\Rightarrow f_x\, dx + f_y\, dy = 0 \Rightarrow f_x = -9x^2+2xy \Rightarrow$ f(x,y) = $-3x^3 + x^2y + \phi(y) \Rightarrow$

$f_y = x^2 + \phi`(y) = 2y+x^2+1 \Rightarrow \phi`(y) = 2y + 1 \Rightarrow \phi(y) = y^2 + y + c$

1-d) The characteristic equation is $r^2+5r+4=0$, therefore r=-4, -1, thus y_c = ($c_1 e^{-4x} + c_2 e^{-x}$)

$$Y_P = \frac{1}{D^2+5D+4}e^{5x}\cos 2x = e^{5x}\frac{1}{(D+5)^2+5(D+5)+4}\cos 2x = e^{5x}\frac{1}{D^2+15D+54}\cos 2x$$

$= \quad e^{5x}\frac{1}{15D+50}\cos 2x = e^{5x}\frac{(15D-50)}{225D^2+2500}\cos 2x = e^{5x}\frac{(-30\sin 2x - 50\cos 2x)}{225(-4)+2500}$

Answer of Q2

Put $y' = z$, hence $y'' = z' = x^3(z+y) = f(x, y, z)$, such that $x_0 = 0$, $y_0 = 1$, $y'(0) = z_0 = ½$.

According to the following two formulas, we can get the first two approximations

$$y_{n+1} = y_0 + \int_{x_0}^{x} z_n\,dx \;, \quad z_{n+1} = z_0 + \int_{x_0}^{x} f(x, y_n, z_n)\,dx = \int_{x_0}^{x} x^3(y_n + z_n)\,dx$$

Put n=0 to obtain y_1, z_1 such that $y_1 = y_0 + \int_{x_0}^{x} z_0\,dx = 1 + \int_0^x \frac{dx}{2} = 1 + \frac{x}{2}$ and

$$z_1 = z_0 + \int_{x_0}^{x} f(x, y_0, z_0)\,dx = \frac{1}{2} + \int_{x_0}^{x} x^3(y_0 + z_0)\,dx = \frac{1}{2} + \int_0^x x^3\left(1 + \frac{1}{2}\right)dx = \frac{1}{2} + \frac{3x^4}{8}$$

Put n=1 to obtain y_2, such that $y_2 = y_0 + \int_{x_0}^{x} z_1\,dx = 1 + \int_0^x \left(\frac{3x^4}{8} + \frac{1}{2}\right)dx = 1 + \frac{x}{2} + \frac{3x^5}{40}$

By using Euler method, states:

$y_{n+1} = y_n + h\,z_n$, $h = 0.1$ and $z_{n+1} = z_n + 0.1\;x_n^3(y_n + z_n)$

$y_1 = y_0 + h\,z_0 = 1 + 0.1(1/2) = 1.05$ and $z_1 = z_0 + 0.1\;x_0^3(y_0 + z_0) = ½$

$y_2 = y_1 + h\,z_1 = 1.05 + 0.1(1/2) = 1.1 = y(0.2)$

Answer of Q3

a) $F(s) = \frac{s\,e^{-2s}}{s^2+6s+20} + \frac{1}{(3s+4)^3} = \frac{(s+3-3)\,e^{-2s}}{(s+3)^2+11} + \frac{1}{27(s+4/3)^3}$, therefore

$$f(t) = e^{-3(t-2)}\left[\cos\sqrt{11}(t-2) - \frac{3}{\sqrt{11}}\sin\sqrt{11}(t-2)\right]U(t-2) + \frac{1}{54}e^{-(4/3)t}t^2$$

b) $L\{\,t\sin 2t\cosh 3t\} = L\{t\sin 2t(e^{3t} + e^{-3t})/2\} = \frac{2(s-3)}{[(s-3)^2+4]^2} + \frac{2(s+3)}{[(s+3)^2+4]^2}$

$$L\{e^{-3t}\int_{u=0}^{t} e^{-2u}\sin u\,du\} = \frac{1}{[(s+5)^2+1][s+3]}$$

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INTENDED LEARNING OUTCOMES OF COURSE (ILOs)

a. Knowledge and Understanding:

a.1) Recognize concepts and theories of mathematics. (A.1)

a.2) Recognize methodologies of solving engineering problems. (A.5)

b. **Intellectual Skills**

b.1) Select appropriate mathematical methods for analyzing problems. (B.1)

b.2) Solve engineering problems, often on the basis of limited and possibly contradicting information (B.7)

c. Professional and Practical Skills

c.1) Apply knowledge, skills of mathematics essential in commercial and industrial environment. (C.15)

d. General and Transferable Skills

d.1) Collaborate effectively within multidisciplinary team. (D.1)

d.2) Lead and motivate individuals. (D.5)

Questions	**Total marks**	**Achieved ILOS**
Q1	**20**	**a1,a2, b1**
Q2	**20**	**a1,b1,b2**