Benha University Faculty of Engineering- Shoubra Electrical Engineering Department First Year Communication(تخلفات)		Final Term Exam Date: 23/5/2016 Engineering Mathematics(2)**B** Duration : **3 hours**
Answer all the following questions	No. of questions : **Two**	Total Mark: **80**

Question 1 **[45 marks]**

(a) Evaluate: (i) $\int_0^{\frac{\pi}{2}} \sqrt{\sin x}\, dx$ (ii) $\int_0^1 \frac{dx}{\sqrt[3]{1-x^3}}$ (iii) $\int_0^\infty \frac{\sin t}{t} dt$ **[15 marks]**

(b) Prove $f(z) = \cos z$ is differentiable at any point then find $f'(z)$ **[10 marks]**

(c) Show that $u(x,y) = \cos x \cosh y$ is harmonic and find $v(x,y)$ such that $f(z) = u + iv$ is analytic , Express $f(z)$ in terms of Z only **[10 marks]**

(d) Evaluate the following integrals: **[10 marks]**

$(i) \int_c \frac{\sinh z}{z-\pi} dz$; $(ii) \int_c \frac{1}{z^2 - 36} dz$;

Where C for all is the circle $|z-1| = 1$

Question 2 **[35 marks]**

(a) Find Laplace transform for **[9 marks]**

$$f(t) = \cos(3t + \frac{\pi}{2}) + t\cosh 2t + e^t \sin 4t$$

(b) If $F(s) = \frac{1}{S^2 + S} + \frac{1}{S^2 + 2S + 17}$ find $f(t)$ **[6 marks]**

(c) Solve the following differential equation using Laplace transform: **[10 marks]**

$$y'' + 2y' + y = 0 ; \quad y(0) = 0 \; ; y'(0) = 1$$

(d) Find the series solution of

$$y'' + xy' = x^2 + 2$$

[10 marks]

Good Luck *Dr.Eng. Zaki Ahmed Zaki*

First Year Comm.(تخلفات) Final term Exam (May 2016)
Model Answer

Question1 (a)

$$(i)\int_0^{\frac{\pi}{2}}\sqrt{\sin x}\,dx=\int_0^{\frac{\pi}{2}}\sin^{\frac{1}{2}}x\cos^0 x\,dx=\frac{1}{2}\beta(\frac{3}{4},\frac{1}{2})=\frac{1}{2}\frac{\Gamma(\frac{1}{2})\Gamma(\frac{3}{4})}{\Gamma(\frac{5}{4})}=\frac{\sqrt{\pi}}{2}\frac{\Gamma(\frac{3}{4})}{\frac{1}{4}\Gamma(\frac{1}{4})}=2\sqrt{\pi}\frac{\Gamma(\frac{3}{4})}{\Gamma(\frac{1}{4})}$$

$$(ii)\int_0^1\frac{dx}{\sqrt[3]{1-x^3}};let x^3=y,x=y^{\frac{1}{3}},dx=\frac{1}{3}y^{\frac{-2}{3}}dy,\frac{1}{3}\int_0^1 y^{\frac{-2}{3}}(1-y)^{\frac{-1}{3}}dy$$

$$=\frac{1}{3}\beta(\frac{1}{3},\frac{2}{3})=\frac{1}{3}\frac{\pi}{\sin\frac{\pi}{3}}=\frac{2\pi}{3\sqrt{3}}$$

$$(iii)\int_0^\infty\frac{\sin t}{t}e^{-t}dt;L(\frac{\sin t}{t})=\int_s^\infty\frac{1}{s^2+1}ds=\frac{\pi}{2}-\tan^{-1}s\to s=0,\int_0^\infty\frac{\sin t}{t}dt=\frac{\pi}{2}-\tan^{-1}0=\frac{\pi}{2}$$

(b)

$f(z)=\cos z=\cos(x+iy)=\cos x\cosh y-i\sin x\sinh y,u=\cos x\cosh y,u_x=-\sin x\cosh y,u_y=\cos x\sinh y$

$v=-\sin x\sinh y,v_x=-\cos x\sinh y,v_y=-\sin x\cosh y;\therefore u_x=v_y\,and\,u_y=-v_x\to f(z)\,is analytic\to f'(z)=-\sin z$

(c)

$u=\sin x\cosh y,u_x=\cos x\cosh y,u_{xx}=-\sin x\cosh y,u_y=\sin x\sinh y,u_{yy}=\sin x\cosh y\to u_{xx}+u_{yy}=0\to harmonic;$

$u_x=v_y\to v=\int u_x dy=\cos x\sinh y+c(x),u_y=-v_x\to\sin x\cosh y=\sin x\cosh y-c'(x)\to$

$c(x)=cons.\to v=\cos x\sinh y+cons.\to f(z)=\sin x\cosh y+i(\cos x\sinh y+cons.)\to$

$let x=z,y=o\to f(z)=\sin z\to f'(z)=\cos z$

(d)

$$(i)\int_c\frac{\sinh z}{z-\pi}dz\ ;\qquad (ii)\int_c\frac{1}{z^2-36}dz\ ;$$

$$(i)\int_c\frac{\sinh z}{z-\pi}dz=0\quad as\ z=\pi\ is\ outside\ the\ region\ |z-1|=1$$

$$(ii)\int_c\frac{1}{z^2-36}dz=0\quad as\ z=\pm 6\ are\ outside\ the\ region\ |z-1|=1$$

Qustion2 (a)

$$f(t)=\cos(3t+\frac{\pi}{2})+t\cosh 2t+e^{t}\sin 4t=-\sin 3t+t\cosh 2t+e^{t}\sin 4t$$

$$F(s)=\frac{-3}{s^{2}+9}+(-1)^{1}(\frac{s}{s^{2}-4})'+\frac{4}{(s-1)^{2}+16}=\frac{-3}{s^{2}+9}+\frac{s^{2}+4}{(s^{2}-4)^{2}}+\frac{4}{(s-1)^{2}+16}$$

(b)

$$F(s)=\frac{1}{S^{2}+S}+\frac{1}{S^{2}+2S+17}=\frac{1}{S(s+1)}+\frac{1}{(S+1)^{2}+4^{2}}=\frac{1/(s+1)}{S}+\frac{1}{3}\frac{3}{(S+1)^{2}+4^{2}}$$

$$f(t)=\int_{0}^{t}e^{-t}\,dt+\frac{1}{3}e^{-t}\sin 3t=(1-e^{-t})+\frac{1}{3}e^{-t}\sin 4t$$

(c) by using Laplace to both sides we get

$$y''+2y'+y=e^{-t}\ ;\quad y(0)=0\ ;\ y'(0)=1$$

$$s^{2}Y(s)-sy(0)-y'(0)+2sY(s)-2y(0)+Y(s)=0$$

$$Y(s)(s^{2}+2s+1)=1\quad then\ Y(s)=\frac{1}{(s+1)^{2}}\rightarrow y(t)=te^{-t}$$

(d)

$x=x_{0}$ *is regular for* $y''+p(x)y'+q(x)y=0;$

if $p(x)$ *and* $q(x)$ *are defined* $(ANALYTIC)$ *at* $x=x_{0}$

$$\rightarrow y=\sum_{n=0}^{\infty}a_{n}(x-x_{0})^{n},$$

$$p(x)=x\,;q(x)=0\rightarrow defined\ at\ x=0\rightarrow$$

$$y=\sum_{n=0}^{\infty}a_{n}x^{n},\quad y'=\sum_{n=1}^{\infty}na_{n}x^{n-1},\quad y''=\sum_{n=2}^{\infty}n(n-1)a_{n}x^{n-2},$$

$$\sum_{n=2}^{\infty}n(n-1)a_{n}x^{n-2}+\sum_{n=1}^{\infty}na_{n}x^{n}=x^{2}+2,$$

let $n-2=m$ *in first sum*, $n=m$ *in second sum*

$$\rightarrow 2a_{2}+\sum_{m=1}^{\infty}((m+2)(m+1)a_{m+2}+ma_{m})x^{m}=x^{2}+2\rightarrow 2a_{2}=2\rightarrow a_{2}=1,$$

$$R.R\ a_{m+2}=\frac{-ma_{m}}{(m+2)(m+1)};m=1,3,4..$$

$$12a_{4}+2a_{2}=1\rightarrow a_{4}=-1/12\rightarrow a_{3}=-a_{1}/6\rightarrow a_{5}=a_{1}/40$$

$$y(x)=a_{0}+a_{1}[x-x^{3}/6+x^{5}/40+....]+[x^{2}-x^{4}/12+...]$$